$F(\lambda,\theta) =$ (27)

$$\prod_{n=1}^{\infty}(1+2\lambda q^n\cos\theta+\lambda^2q^{2n}) = (-\lambda e^{i\theta}q;q)_\infty(-\lambda e^{-i\theta}q;q)_\infty$$

$$= \sum_{r=0}^{\infty}\frac{(\lambda e^{i\theta})^r q^{r(r+1)/2}}{(q)_r}\sum_{s=0}^{\infty}\frac{(+\lambda e^{-i\theta})^s q^{s(s+1)/2}}{(q)_s}$$

$$= \sum_{r,s\geq 0}\frac{\lambda^{r+s}e^{i\theta(r-s)}q^{(r+s+1)(r+s)/2}}{(q)_r(q)_s}\cdot q^{-rs}$$

Using q-Chu-Van:

$$\sum_{j\geq 0}\frac{(a)_j(q^{-s})_j q^j}{(c)_j(q)_j} = \frac{a^s(c/a)_s}{(c)_s}$$

with $c=0$ & $a=q^{-r}$:

$$\sum_{j\geq 0}\frac{(+q^{-r})_j(q^{-s})_j q^j}{(q)_j} = q^{-rs}.$$

Hence,

$$F(\lambda,\theta) = \sum_{r,s\geq 0}\frac{\lambda^{r+s}e^{i\theta(r-s)}q^{(r+s+1)(r+s)/2}}{(q)_r(q)_s}\sum_{j\geq 0}\frac{(q^{-r})_j(q^{-s})_jq^j}{(q)_j}$$

$$= \sum_{r,s\geq 0}\sum_{j=0}^{\min(r,s)}\frac{\lambda^{r+s}e^{i\theta(r-s)}q^{(r+s+1)(r+s)/2}q^j q^{j(j-1)-(r+s)j}}{(q)_{r-j}(q)_{s-j}(q)_j}$$

$(r\to r+j,\ s\to s+j)$