$$= \sum_{j=0}^{\infty} \sum_{r,s\geq 0} \frac{\lambda^{r+s+2j} e^{i\theta(r-s)} q^{(r+s+j)(r+s+j-1)/2}}{(q)_r (q)_s (q)_j} \cdot q^{(j+1)j/2+r+s+j} \quad (28)$$

$$= \sum_{j,n\geq 0} \sum_{\substack{r,s\geq 0 \\ r+s=n}} \left(\quad \right)$$

$$= \sum_{j,n\geq 0} \frac{\lambda^{n+2j} q^{(n+j)(n+j-1)/2+(j+1)j/2+n+j}}{(q)_n \cancel{(q)_r (q)_s} (q)_j} \cdot \sum_{\substack{r,s\geq 0 \\ r+s=n}} \frac{(q)_n}{(q)_r (q)_s} e^{i\theta(r-s)}$$

$$= \sum_{n=0}^{\infty} \frac{\lambda^n}{(q)_n} q^{n(n+1)/2} A_n(\theta) \sum_{j=0}^{\infty} \frac{\lambda^{2j} q^{j^2+(n+1)j}}{(q)_j}$$

We need JTP

$$(z)_\infty (q/z)_\infty (q)_\infty = \sum_{n=-\infty}^{\infty} (-1)^n z^n q^{n(n-1)/2}$$

$$F\left(\frac{1}{\sqrt{q}}, \theta\right) = (-\sqrt{q}\, e^{i\theta}; q)_\infty (-\sqrt{q}\, e^{-i\theta}; q)$$

$$= \frac{1}{(q)_\infty} \sum_{n=-\infty}^{\infty} (\sqrt{q})^n e^{ni\theta} q^{n(n-1)/2} \qquad (z = -\sqrt{q}\, e^{i\theta})$$

$$= \frac{1}{(q)_\infty} \sum_{n=-\infty}^{\infty} q^{n^2/2} e^{in\theta} = \frac{1}{(q)_\infty} \left(1 + 2\sum_{n=1}^{\infty} \cos(n\theta)\, q^{n^2/2}\right)$$