<u>Bailey's Lemma</u> If for $n \geq 0$

$$(*) \qquad \beta_n = \sum_{r=0}^{n} \frac{\alpha_r}{(q)_{n-r}\,(aq)_{n+r}}$$

then

$$\beta_n' = \sum_{r=0}^{n} \frac{\alpha_r'}{(q)_{n-r}\,(aq)_{n+r}}$$

where

$$\alpha_r' = \frac{(\rho_1)_r (\rho_2)_r (aq/\rho_1\rho_2)^r \alpha_r}{(aq/\rho_1)_r\,(aq/\rho_2)_r}$$

&

$$\beta_n' = \sum_{j=0}^{n} \frac{(\rho_1)_j (\rho_2)_j (aq/\rho_1\rho_2)_{n-j} \left(\frac{aq}{\rho_1\rho_2}\right)^j \beta_j}{(q)_{n-j}\,\left(\frac{aq}{\rho_1}\right)_n \left(\frac{aq}{\rho_2}\right)_n}$$

<u>Note</u>: A pair of sequences (α_n, β_n) related by $(*)$ is called a Bailey pair. So (α_n', β_n') is a new Bailey pair.

<u>Proof</u>: We apply the Bailey transform with

$$\delta_n = \frac{(\rho_1)_n (\rho_2)_n \,(q^{-N})_n\, q^n}{(\rho_1\rho_2 q^{-N}/a)_n}$$

$$u_n = \frac{1}{(q)_n} \qquad v_n = \frac{1}{(aq)_n}.$$

We need q-Pfaff-Saalschütz:

$$\sum_{j=0}^{N} \frac{(a)_j (b)_j (q^{-N})_j\, q^j}{(c)_j (abq^{1-N}/c)_j (q)_j} = \frac{(c/a)_N (c/b)_N}{(c)_N (c/ab)_N}.$$