$$\gamma_n = \sum_{r=n}^{\infty} \delta_r \, u_{r-n} \, v_{r+n}$$

$$= \sum_{r=n}^{\infty} \frac{(\rho_1)_r (\rho_2)_r (q^{-N})_r \, q^r}{(\rho_1\rho_2 q^{-N}/a)_r \, (q)_{r-n} \, (aq)_{n+r}}$$

$$= \sum_{r=0}^{\infty} \frac{(\rho_1)_{r+n} (\rho_2)_{r+n} (q^{-N})_{r+n} \, q^{r+n}}{(\rho_1\rho_2 \frac{q^{-N}}{a})_{r+n} \, (q)_r \, (aq)_{r+2n}}$$

$$\left(\text{now} \quad (a)_{r+n} = (a)_r (q^r a)_n = (a)_n (q^n a)_r\right)$$

$$= \frac{(\rho_1)_n (\rho_2)_n (q^{-N})_n \, q^n}{(\rho_1\rho_2 q^{-N}/a)_n \, (aq)_{2n}} \sum_{r=0}^{\infty} \frac{(q^n\rho_1)_r (q^n\rho_2)_r (q^{-(N-n)})_r \, q^r}{(\rho_1\rho_2 q^{-N+n}/a)_r \, (aq^{2n+1})_r \, (q)_r}$$

$$= \frac{(\rho_1)_n (\rho_2)_n (q^{-N})_n \, q^n}{(\rho_1\rho_2 q^{-N}/a)_n \, (aq)_{2n}} \; \frac{(aq^{n+1}/\rho_1)_{N-n} \, (aq^{n+1}/\rho_2)_{N-n}}{(aq^{2n+1})_{N-n} \, \left(\frac{aq}{\rho_1\rho_2}\right)_{N-n}}$$

$$\left(\text{Recall,} \quad \frac{(x)_{N-n}}{(x)_N} = \frac{1}{(q^{1-N}/x)_n} \, \frac{1}{(-x q^{N-n})^n q^{n(n-1)/2}}\right)$$

&

$$(xq^{n+1})_{N-n} = (1-xq^{n+1}) \cdots (1-xq^N)$$
$$= \frac{(xq)_N}{(xq)_n}$$

$$= \frac{(aq/\rho_1)_N}{(aq/\rho_1)_n} \, \frac{(aq/\rho_2)_N}{(aq/\rho_2)_n} \, \frac{(\rho_1)_n (\rho_2)_n (q^{-N})_n \left(-\frac{aq^{1+N-n}}{\rho_1\rho_2}\right)^n q^{n(n-1)/2}}{(aq/(\rho_1\rho_2))_N}$$

$$\cdot \frac{q^n}{(aq)_{N+n}}$$