

$$\begin{aligned}
 \gamma_n &= \sum_{r=n}^{\infty} \delta_r u_{r-n} v_{r+n} \\
 &= \sum_{r=n}^{\infty} \frac{(p_1)_r (p_2)_r (q^{-N})_r q^r}{(p_1 p_2 q^{-N}/a)_r (q)_{r-n} (aq)_{n+r}} \\
 &= \sum_{r=0}^{\infty} \frac{(p_1)_{r+n} (p_2)_{r+n} (q^{-N})_{r+n} q^{r+n}}{(p_1 p_2 q^{-N}/a)_{r+n} (q)_r (aq)_{r+2n}}
 \end{aligned}$$

$$\text{(now } (a)_{r+n} = (a)_r (q^r a)_n = (a)_n (q^n a)_r \text{)}$$

$$= \frac{(p_1)_n (p_2)_n (q^{-N})_n q^n}{(p_1 p_2 q^{-N}/a)_n (aq)_{2n}} \sum_{r=0}^{\infty} \frac{(q^n p_1)_r (q^n p_2)_r (q^{-N-n})_r q^r}{(p_1 p_2 q^{-N+n}/a)_r (aq^{2n+1})_r (q)_r}$$

$$= \frac{(p_1)_n (p_2)_n (q^{-N})_n q^n}{(p_1 p_2 q^{-N}/a)_n (aq)_{2n}} \frac{(aq^{n+1}/p_1)_{N-n} (aq^{n+1}/p_2)_{N-n}}{(aq^{2n+1})_{N-n} \left(\frac{aq}{p_1 p_2}\right)_{N-n}}$$

$$\text{(Recall, } \frac{(x)_{N-n}}{(x)_N} = \frac{1}{(q^{N-n}/x)_n} \frac{1}{(-x q^{N-n})_n q^{n(n-1)/2}} \text{)}$$

$$\begin{aligned}
 &\& \\
 (xq^{n+1})_{N-n} &= (1-xq^{n+1}) \cdots (1-xq^N) \\
 &= \frac{(xq)_N}{(xq)_n}
 \end{aligned}$$

$$= \frac{(aq/p_1)_N}{(aq/p_1)_n} \frac{(aq/p_2)_N}{(aq/p_2)_n} \frac{(p_1)_n (p_2)_n (q^{-N})_n \left(-\frac{aq^{1+N-n}}{p_1 p_2}\right)_n q^{n(n-1)/2}}{(aq/(p_1 p_2))_N \left(\frac{aq}{p_1 p_2}\right)_n} q^n$$

$$\frac{q^n}{(aq)_{N+n}}$$