$$(aq)_{N+n} = (aq)_N \, (aq^{N+1})_n$$

so

$$\gamma_n = \frac{(aq/\rho_1)_N \, (aq/\rho_2)_N}{(aq)_N \, (aq/(\rho_1\rho_2))_N} (-1)^n \frac{(\rho_1)_n(\rho_2)_n(q^{-N})_n \left(\frac{aq}{\rho_1\rho_2}\right)^n q^{nN-n(n-1)/2}}{(aq/\rho_1)_n (aq/\rho_2)_n (aq^{N+1})_n}$$

Now, $$\sum_{r=0}^{N} \frac{\alpha_r'}{(q)_{N-r} \, (aq)_{N+r}}$$

$$= \sum_{r=0}^{N} \frac{(\rho_1)_r(\rho_2)_r(aq/\rho_1\rho_2)^r \, \alpha_r}{(aq/\rho_1)_r(aq/\rho_2)_r \, (q)_{N-r}(aq)_{N+r}}$$

$$\left((q)_{N-r} = \frac{(q)_N}{(q^{-N})_r \, (-q^{1+N-r})^r \, q^{r(r-1)/2}} \right)$$

$$= \sum_{r=0}^{N} \frac{(\rho_1)_r(\rho_2)_r \, (q^{-N})_r \, (aq/\rho_1\rho_2)^r(-1)^r}{(aq/\rho_1)_r \, (aq/\rho_2)_r \, (aq)_{N+r} \, (q)_N} q^{rN-r(r-1)/2} \, \alpha_r$$

$$= \frac{(aq/\rho_1\rho_2)_N}{(aq/\rho_1)_N (aq/\rho_2)_N (q)_N} \sum_{r=0}^{N} \gamma_r \, \alpha_r$$

$$= \frac{(aq/\rho_1\rho_2)_N}{(aq/\rho_1)_N(aq/\rho_2)_N} \frac{1}{(q)_N} \sum_{r=0}^{N} \beta_r \, \delta_r \qquad \text{(by Bailey Transf.)}$$

$$= \frac{(aq/\rho_1\rho_2)_N}{(aq/\rho_1)_N(aq/\rho_2)_N} \frac{1}{(q)_N} \sum_{r=0}^{N} \frac{(\rho_1)_r(\rho_2)_r \, (q^{-N})_r \, q^r}{(\rho_1\rho_2 q^{-N}/a)_r} \beta_r$$