$$\frac{(q^{-N})_r}{(q)_N} = \frac{1}{(q)_{N-r}} \frac{1}{(-q^{1+N-r})^r q^{r(r-1)/2}}$$

$$\frac{(x)_N}{(q^{1-N}/x)_r} = (x)_{N-r} (-x q^{N-r})^r q^{r(r-1)/2}$$

Hence
$$\sum_{r=0}^{N} \frac{\alpha_r'}{(q)_{N-r} (aq)_{N+r}}$$

$$= \sum_{r=0}^{N} \frac{(\rho_1)_r (\rho_2)_r}{(q)_{N-r}} \frac{(aq/\rho_1\rho_2)_{N-r}}{(aq/\rho_1)_N (aq/\rho_2)_N} \left(\frac{aq}{\rho_1\rho_2}\right)^r \beta_r$$

$$= \beta_N'. \qquad \square$$

<u>Theorem</u> If (α_n, β_n) is a Bailey pair then

$$\sum_{j\geq 0} a^j q^{j^2} \beta_j = \frac{1}{(aq)_\infty} \sum_{r=0}^{\infty} a^r q^{r^2} \alpha_r.$$

<u>Proof</u> $(\rho_1)_r \frac{1}{\rho_1^r} = (1-\rho_1)(1-\rho_1 q) \cdots (1-\rho_1 q^{r-1}) \frac{1}{\rho_1^r}$

$= (\frac{1}{\rho_1}-1)(\frac{1}{\rho_1}-q) \cdots (\frac{1}{\rho_1}-q^{r-1})$

$$\lim_{\rho_1\to\infty} (\rho_1)_r \rho_1^{-r} = (-1)^r q^{r(r-1)/2}$$

$$\lim_{\rho_1,\rho_2\to\infty} \alpha_r' = a^r q^{r^2} \alpha_r$$