

$$\lim_{p_1, p_2 \rightarrow \infty} \beta_n' = \sum_{j=0}^{\infty} \frac{a^j q^{j^2}}{(q)_{n-j}} \beta_j$$

Done by Bailey's lemma,

$$\sum_{j=0}^n \frac{a^j q^{j^2}}{(q)_{n-j}} \beta_j = \sum_{r=0}^n \frac{a^r q^{r^2}}{(q)_{n-r} (aq)_{n+r}}$$

Letting $n \rightarrow \infty$ (& assuming certain convergence conditions)

$$\frac{1}{(q)_{\infty}} \sum_{j=0}^{\infty} a^j q^{j^2} \beta_j = \sum_{r=0}^{\infty} \frac{a^r q^{r^2}}{(q)_{\infty} (aq)_{\infty}}$$

and

$$\sum_{j=0}^{\infty} a^j q^{j^2} \beta_j = \frac{1}{(aq)_{\infty}} \sum_{r=0}^{\infty} a^r q^{r^2} \alpha_r.$$

Cor If (α_n, β_n) is a Bailey pair then

(α_n', β_n') is a Bailey pair

where

$$\alpha_n' = \sum_{r=0}^{\infty} a^r q^{r^2} \alpha_r$$

$$\beta_n' = \sum_{j=0}^n \frac{a^j q^{j^2}}{(q)_{n-j}} \beta_j.$$