$(r = n + j, \quad j = r - n)$

$$= \sum_{r=0}^{2n} \frac{(-1)^{r-n} q^{(r-n)(r-n-1)/2}}{(q)_r (q)_{2n-r}}$$

$$= \frac{(-1)^n}{(q)_{2n}} \sum_{r=0}^{2n} (-1)^r q^{r(r-1)/2 - rn + n(n+1)/2} \begin{bmatrix} 2n \\ r \end{bmatrix}$$

$$= \frac{(-1)^n q^{n(n+1)/2}}{(q)_{2n}} (q^{-n})_{2n} = 0$$

$$\cancel{= 1 \text{ if } n = 0}$$

since $n \geq 1$ & $(q^{-n})_{2n} = (1-q^{-n}) \cdots (1-q^{0}) \cdots (1-q^{n-1}) = 0.$

$\square$

Hence

$$\alpha_n' = q^{n^2} \alpha_n = \begin{cases} 1 & \text{if } n = 0 \\ (-1)^n q^{n(3n-1)/2} (1+q^n), & n \geq 1 \end{cases}$$

$$\beta_n' = \sum_{j=0}^{n} \frac{q^j q^{j^2} \beta_j}{(q)_{n-j}} = \frac{1}{(q)_n}$$

form a Bailey pair.