Proof: $\sum_{n=0}^{\infty} A(n) q^n = \prod_{n=0}^{\infty} (1+q^{3n+1})(1+q^{3n+2})$

$$= \prod_{n=0}^{\infty} \frac{(1-q^{6n+2})(1-q^{6n+4})}{(1-q^{3n+1})(1-q^{3n+2})}$$

$$= \frac{\displaystyle\prod_{\substack{n\geq 1 \\ n\equiv 2,4 \pmod 6}} (1-q^n)}{\displaystyle\prod_{\substack{n\geq 1 \\ n\equiv 1,2 \pmod 3}}}$$

$$= \frac{\displaystyle\prod_{\substack{n\geq 1 \\ n\equiv 2,4 \pmod 6}} (1-q^n)}{\displaystyle\prod_{\substack{n\geq 1 \\ n\equiv 1,2,4,5 \pmod 6}} (1-q^n)} = \prod_{\substack{n\geq 1 \\ n\equiv 1,5 \\ \pmod 6}} \frac{1}{1-q^n}$$

$$= \sum_{n=0}^{\infty} B(n) q^n.$$

Hence $A(n) = B(n)$ for all n.