$$\alpha_1 = f_5(q) = 1+q+q^2+q^3+q^4+2q^5+q^6+q^7$$

uniquely determine α_n for $n \geq 0$.

<u>Note:</u> We want hold (*) for $m=1$.

$$\alpha_1 = 1+q+q^2+q^3+q^4+2q^5+q^6+q^7$$
$$= (1+q^4+q^5)\alpha_0 + q^3(1-q^3)\alpha_{-1}$$

$$q^3(1-q^3)\alpha_{-1} = 1+q+q^2+q^3+q^4+2q^5+q^6+q^7 - (1+q^4+q^5)(1+q+q^2)$$
$$= q^3 - q^6$$

and need $\alpha_{-1} = 1$.

Now, for $|z|<1$ & $|q|<1$ let

$$g(z,q) := \sum_{n=0}^{\infty} \delta_n(q)\, z^n = \frac{(-zq;q^3)_\infty(-zq^2;q^3)_\infty}{(z;q^3)_\infty}$$

Then

$$g(zq^3,q) = \frac{(-zq^4;q^3)_\infty(-zq^5;q^3)_\infty}{(zq^3;q^3)_\infty}$$
$$= g(z,q)\frac{(1-z)}{(1+zq)(1+zq^2)}$$

$$(1-z)\,g(z,q) = (1+zq+zq^2+z^2q^3)\,g(zq^3,q)$$

$$(1-z)\sum_{n=0}^{\infty} \delta_n(q)\, z^n = (1+zq+zq^2+z^2q^3)\sum_{n=0}^{\infty} z^n q^{3n}\delta_n(q)$$