Chapter 6 Sums of Squares

When $n \geq 0$, $(a)_n = (1-a)(1-aq)\cdots(1-aq^{n-1})$

$$= \frac{(a)_\infty}{(aq^n)_\infty}.$$

When $n = -m < 0$ we define

$$(a)_n := \frac{(a)_\infty}{(aq^n)_\infty} = \frac{(a)_\infty}{(aq^{-m})_\infty} = \frac{1}{(1-aq^{-m})\cdots(1-aq^{-1})}$$

$$= \frac{(-q/a)^m \, q^{m(m-1)/2}}{(q/a)_m}.$$

Bilateral Basic Series:

$${}_r\psi_r\left[\begin{matrix} a_1, a_2, \ldots a_r, \\ b_1, b_2, \ldots, b_r \end{matrix}; q, z\right] := \sum_{n=-\infty}^{\infty} \frac{(a_1)_n (a_2)_n \cdots (a_r)_n}{(b_1)_n (b_2)_n \cdots (b_r)_n} z^n$$

$$= \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \cdots (a_r)_n \, z^n}{(b_1)_n (b_2)_n \cdots (b_r)_n}$$

$$+ \sum_{n=1}^{\infty} \frac{(q/b_1)_n (q/b_2)_n \cdots (q/b_r)_n}{(q/a_1)_n (q/a_2)_n \cdots (q/a_r)_n} \left(\frac{b_1 b_2 \cdots b_r}{a_1 a_2 \cdots a_r z}\right)^n$$

converges for $|q| < 1$ and

$$\left|\frac{b_1 b_2 \cdots b_r}{a_1 a_2 \cdots a_r}\right| < |z| < 1$$