<u>Corollary (Lagrange)</u>
Every integer $n \geq 1$ can be written as the sum of four squares.

<u>Application (2)</u>
Let $b, c, d, e = -1$ and let $a \to 1$ we find (eventually) that

<u>Theorem</u> (Jacobi) For $n \geq 1$,

$$r_8(n) = 16(-1)^n \sum_{d|n} (-1)^d d^3.$$

<u>Application (3)</u>
Let $q \to q^5$, $a = q^4$, $b = c = q$ $d = e = q^3$.
We find (eventually) that

$$\sum_{n=0}^{\infty} \frac{q^{5n+1}}{(1-q^{5n+1})^2} - \frac{q^{5n+2}}{(1-q^{5n+2})^2} - \frac{q^{5n+3}}{(1-q^{5n+3})^2} + \frac{q^{5n+4}}{(1-q^{5n+4})^2}$$

$$= q \frac{(q^5; q^5)_\infty^5}{(q)_\infty} \quad \text{(Ramanujan)}$$