(16)

Corollary $\displaystyle\sum_{n=0}^{\infty} p(5n+4)q^n = 5\prod_{n=1}^{\infty}\frac{(1-q^{5n})^5}{(1-q^n)^6}.$

Proof $$\sum_{n=0}^{\infty} p(n)q^n = \frac{1}{(q)_\infty} = \frac{(q^5;q^5)_\infty^5}{(q)_\infty}\cdot\frac{1}{(q^5;q^5)_\infty^5}$$

Hence
$$\sum_{n=0}^{\infty} p(5n+4)q^{5n+4} = \frac{1}{(q^5;q^5)_\infty^5}\sum_{n=0}^{\infty} a_5(5n+4)q^{5n+4}$$

$$\sum_{n=0}^{\infty} p(5n+4)q^{5n} = \frac{1}{(q^5;q^5)_\infty^5}\sum_{n=0}^{\infty} a_5(5n+4)q^{5n}$$

$$\sum_{n=0}^{\infty} p(5n+4)q^{n} = \frac{1}{(q)_\infty^5}\sum_{n=0}^{\infty} a_5(5n+4)q^{n}$$

$$= \frac{1}{(q)_\infty^5}\, 5\sum_{n=0}^{\infty} a_5(n)q^n$$

$$= \frac{5}{(q)_\infty^5}\,\frac{(q^5;q^5)_\infty^5}{(q)_\infty}$$

$$= 5\,\frac{(q^5;q^5)_\infty^5}{(q)_\infty^6} = 5\prod_{n=1}^{\infty}\frac{(1-q^{5n})^5}{(1-q^n)^6}. \quad \square$$