**Ramanujan's ${}_1\psi_1$ summation** For $|q|<1$,

$$(*)\quad {}_1\psi_1\left(\begin{matrix}a\\ b\end{matrix};q,z\right)=\sum_{n=-\infty}^{\infty}\frac{(a)_n}{(b)_n}z^n=\frac{(\frac{b}{a})_\infty(az)_\infty(\frac{q}{az})_\infty(q)_\infty}{(b)_\infty(q/a)_\infty(\frac{b}{az})_\infty(z)_\infty}$$

for $\left|\frac{b}{a}\right|<|z|<1$.

We need

**Lemma** If $f(z)$ is analytic for $|z|<1$
and $f(a_n)=0$ for $n\geq 1$
~~where $\lim_{n\to\infty} a_n$~~ for infinitely many $a_n$ when $a_n\to 0$ as $n\to\infty$,
then $f(z)=0$ for all $|z|<1$.

We need $q$-bin Thm:

$$\sum_{n=0}^{\infty}\frac{(a)_n}{(q)_n}t^n=\frac{(at)_\infty}{(t)_\infty}\qquad \text{for } |q|<1,\ |t|<1.$$

**Proof of $(*)$** We show that $(*)$ holds for $b=q^N$
where $N$ is a positive integer.

$${}_1\psi_1\left(\begin{matrix}a\\ b\end{matrix};q,z\right)=\sum_{n=0}^{\infty}\frac{(a)_n}{(b)_n}z^n+\sum_{n=1}^{\infty}\frac{(q/b)_n}{(q/a)_n}\left(\frac{b}{az}\right)^n$$

$${}_1\psi_1\left(\begin{matrix}a\\ q^N\end{matrix};q,z\right)=\sum_{n=0}^{\infty}\frac{(a)_n}{(q^N)_n}z^n+\sum_{n=1}^{\infty}\frac{(q^{1-N})_n}{(q/a)_n}\left(\frac{b}{az}\right)^n$$