$$= \sum_{n=0}^{\infty} \frac{(a)_n z^n}{(q^N)_n} + \sum_{n=1}^{N-1} \frac{(q^{1-N})_n}{(q/a)_n} \left(\frac{b}{az}\right)^n$$

(since $(q^{1-N})_n = 0$ if $n \geq N$)

$$= \sum_{n=1-N}^{\infty} \frac{(a)_n z^n}{(q^N)_n}$$

$$= \sum_{n=0}^{\infty} \frac{(a)_{n+1-N}}{(q^N)_{n+1-N}} z^{n+1-N}$$

Now $(a)_{n+1-N} = \dfrac{(a)_\infty}{(aq^{n+1-N})_\infty} = \dfrac{(a)_\infty}{(aq^{1-N})_\infty} \dfrac{(aq^{1-N})_\infty}{(aq^{1-N+n})_\infty}$

$$= (a)_{1-N} (aq^{1-N})_n,$$

and

$$(q^N)_{n+1-N} = (q^N)_{1-N} (q)_n .$$

Hence

$${}_1\psi_1\left(\begin{matrix} a \\ q^N \end{matrix}; q, z\right) = \frac{(a)_{1-N} z^{1-N}}{(q^N)_{1-N}} \sum_{n=0}^{\infty} \frac{(aq^{1-N})_n z^n}{(q)_n}$$

$$= \frac{(a)_{1-N} z^{1-N}}{(q^N)_{1-N}} \cdot \frac{(azq^{1-N})_\infty}{(z)_\infty} \qquad \text{(by } q\text{-bin Thm.)}$$