$n \pmod 4$	$i^n + i^{-n}$	
0	2	$i^2 = -1$
1	$i + i^{-1} = 0$	$i = -i^{-1}$
2	-2	
3	$i^3 + i^{-3} = 0$	

So $i^{2m} + i^{-2m} = (-1)^m 2$

So ${}_1\psi_1\left(\begin{matrix}-1\\-q\end{matrix}; q, i\sqrt{q}\right) =$

$$= 1 + \sum_{n=1}^{\infty} \frac{(1+1)\cdots(1+q^{n-1})}{(1+q)\cdots(1+q^n)}(i^n + i^{-n}) q^{n/2}$$

$$= 1 + 2\sum_{n=1}^{\infty} \frac{(i^n + i^{-n}) q^{n/2}}{1+q^n}$$

$$= 1 + 4\sum_{n=1}^{\infty} \frac{q^n}{1+q^{2n}}$$

By Ramanujan's summation,

$$1 + 4\sum_{n=1}^{\infty} \frac{(-1)^n q^n}{1+q^{2n}} = \frac{(q)_\infty (-i\sqrt{q})_\infty \left(\frac{q}{-i\sqrt{q}}\right)_\infty (q)_\infty}{(-q)_\infty (-q)_\infty \left(\frac{-q}{-i\sqrt{q}}\right)_\infty (i\sqrt{q})_\infty}$$

$$= \frac{(q)_\infty^2 (-i\sqrt{q})_\infty (i\sqrt{q})_\infty}{(-q)_\infty^2 (i\sqrt{q})_\infty (-i\sqrt{q})_\infty} = \left(\frac{(q)_\infty}{(-q)_\infty}\right)^2$$