Bailey's ${}_6\psi_6$ Summation (1936)

$$
{}_6\psi_6\left[\begin{matrix} q\sqrt{a}, -q\sqrt{a}, b, c, d, e \\ \sqrt{a}, -\sqrt{a}, \frac{aq}{b}, \frac{aq}{c}, \frac{aq}{d}, \frac{aq}{e} \end{matrix}; q, \frac{qa^2}{bcde}\right]
$$

$$
= \frac{(aq)_\infty \left(\frac{aq}{bc}\right)_\infty \left(\frac{aq}{bd}\right)_\infty \left(\frac{aq}{be}\right)_\infty \left(\frac{aq}{cd}\right)_\infty \left(\frac{aq}{ce}\right)_\infty \left(\frac{aq}{de}\right)_\infty (q)_\infty \left(\frac{q}{a}\right)_\infty}{\left(\frac{aq}{b}\right)_\infty \left(\frac{aq}{c}\right)_\infty \left(\frac{aq}{d}\right)_\infty \left(\frac{aq}{e}\right)_\infty \left(\frac{q}{b}\right)_\infty \left(\frac{q}{c}\right)_\infty \left(\frac{q}{d}\right)_\infty \left(\frac{q}{e}\right)_\infty \left(\frac{qa^2}{bcde}\right)_\infty}
$$

if $\left|\frac{qa^2}{bcde}\right| < 1$, (& $|q|<1$).

Note: $\frac{(q\sqrt{a})_n}{(\sqrt{a})_n} \frac{(-q\sqrt{a})_n}{(-\sqrt{a})_n} = \frac{(1-\sqrt{a}q^n)(1+\sqrt{a}q^n)}{(1-\sqrt{a})(1+\sqrt{a})}$

$= \frac{(1-aq^{2n})}{(1-a)}$ for $n \geq 0$.

$$
{}_6\psi_6[\ ] = \sum_{n=0}^{\infty} \left(\frac{1-aq^{2n}}{1-a}\right) \frac{(b)_n (c)_n (d)_n (e)_n}{\left(\frac{aq}{b}\right)_n \left(\frac{aq}{c}\right)_n \left(\frac{aq}{d}\right)_n \left(\frac{aq}{e}\right)_n} \left(\frac{qa^2}{bcde}\right)^n
$$

$$
+ \sum_{n=1}^{\infty} \left(\frac{1-\frac{1}{a}q^{2n}}{1-1/a}\right) \frac{(b/a)_n \left(\frac{c}{a}\right)_n \left(\frac{d}{a}\right)_n \left(\frac{e}{a}\right)_n}{\left(\frac{q}{b}\right)_n \left(\frac{q}{c}\right)_n \left(\frac{q}{d}\right)_n \left(\frac{q}{e}\right)_n} \left(\frac{qa^2}{bcde}\right)^n
$$

$$
\frac{-c^5 q^4}{bcde} \quad \frac{1}{-q^2 abcde \cdot \frac{qa^2}{bcde}} = \frac{qa^2}{bcde}
$$