

HW 3 - Due Friday, May 20

(1)

- [10pts] (7) Let $p(D, n) = \#$ of partitions of n into distinct parts.
Prove that
- $$p(D, n) < e^{\pi\sqrt{n/3}} \quad \text{for } n \geq 1.$$

HINTS:

(i) ~~Let~~ $F(q) := \sum_{n=0}^{\infty} p(D, n) q^n = \prod_{n=1}^{\infty} (1+q^n) = \prod_{n=0}^{\infty} \frac{1}{1-q^{2n+1}}$.

Let $0 < q < 1$. Show that

$$\log F(q) = \sum_{n=1}^{\infty} \frac{1}{n} \frac{q^n}{1-q^{2n}}$$

- (ii) Show that $\frac{e^x}{e^{2x}-1} < \frac{1}{2x}$ for $x > 0$.

Hence show that $\frac{q^n}{1-q^{2n}} < \frac{1}{2nt}$ where $t = -\ln\left(\frac{1}{q}\right)$

- (iii) Hence show that

$$\log p(D, n) < \frac{\pi^2}{12t} + nt$$

for all $t > 0$. and find the minimum value of $\frac{\pi^2}{12t} + nt$.