

(2)

[5 pts] (8) Let $\mu(n)$ be the Möbius function.
 Prove that

$$(i) \quad e^{\frac{1}{b}} = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{1}{b}\right)^n = \prod_{n=1}^{\infty} \left(1 - \frac{1}{b^n}\right)^{-\mu(n)/n}$$

formally.

$$\text{[Hint: } \sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n=1 \\ 0 & \text{if } n>1 \end{cases}$$

[2 pts] (ii) Check this result in MAPLE:

> with (numtheory):

> xc := mul((1 - 1/b^n)^(-mobius(n)/n), n=1..20):

> series(x - exp(1/b), b, 20);