HW4 - Due Tuesday, May 31

[10 pts] (9)(a) Give two proofs of the identity

$$(*)\quad \prod_{n=0}^{\infty}(1+zq^{2n+1}) = \sum_{n=0}^{\infty} \frac{z^n q^{n^2}}{(q^2;q^2)_n},$$

as follows:

(i) Let $F(z,q) := \prod_{n=0}^{\infty}(1+zq^{2n+1}) = \sum_{n=0}^{\infty} A_n(q) z^n$

Show that $F(z,q) = (1+zq)\,F(zq^2,q)$

and that

$$A_m = \frac{q^{2m-1}}{1-q^{2m}} A_{m-1} \quad \text{for } m \geq 1.$$

Hence find A_n & deduce the identity.

(ii) For the second proof show that $(*)$ is a special case of Euler's Corollary to the q-binomial Theorem (see (2.2.6), p. 19).

[BONUS] [5pts] (b) A partition λ is self-conjugate if $\lambda' = \lambda$. For example $\lambda = (5,3,2,1,1)$ is self-conjugate.

(i) Show that $\sum_{n=0}^{\infty} q^{n^2}/(q^2;q^2)_n$

is the generating function for self-conjugate partitions by using Durfee squares.