[BONUS 10pts] (e) With the aid of MAPLE show that

$$\prod_{k=0}^{6} \left(X(\zeta^k q) \right) = (y_1 + y_2 + y_3 + 8)^2$$

and deduce that

(i) $y_1 + y_2 + y_3 = -T-8$

(ii) $y_1 y_2 + y_2 y_3 + y_3 y_1 = T+5$

and (iii) $y_1 y_2 y_3 = 1.$

[Bonus 10pts] (f) With the aid of MAPLE (or ?) let

$$\sum_n x_n q^n = \prod_{k=1}^{6} X(\zeta^k q).$$

Find $\sum_n x_{7n} q^{7n}$ in terms of α, β, and γ and hence show that

$$\sum_n x_{7n} q^{7n} = 7T + 49$$

[5pts] (g) $$\sum_{n=0}^{\infty} p(n) q^n = \frac{1}{E(q)} = \frac{1}{q^2 E(q^{49}) X(q)}$$

$$= \frac{1}{q^2 E(q^{49})} \frac{\prod_{k=1}^{6} X(\zeta^k q)}{\prod_{k=0}^{6} X(\zeta^k q)}.$$

Hence show that $\sum_{n=0}^{\infty} p(7n+5) q^{7n+5} = \dfrac{7T^{-1} + 49T^{-2}}{q^2 E(q^{49})}$

and hence obtain the result.