HW6 – Due Thurs June 16

[10pts] (1) Prove the 2^{nd} Rogers–Ramanujan identity
$$\sum_{n=0}^{\infty} \frac{q^{n^2+n}}{(q)_n} = \prod_{n=0}^{\infty} \frac{1}{(1-q^{5n+2})(1-q^{5n+3})}$$
by using $H_1(z) = H_2(zq)$ and letting $z=q$.

[10pts] (2) Prove the 2nd Rogers–Ramanujan identity using Rogers method ie by using
$$a_l = \sum_{h \geq 0} T(l,h)\, b_{l+h}$$
for the function $F(\frac{1}{\sqrt{q}}, \theta)$.

[BONUS 10 pts] (3) Suppose $b_r = \sum_{j \geq 0} \begin{bmatrix} r+2j \\ j \end{bmatrix} a_{r+2j}$ for all $r \geq 0$. Prove (formally) directly that
$$a_r = \sum_{h \geq 0} T(r,h)\, b_{r+h}$$
where
$$T(r,N) = \begin{cases} 1 & \text{if } N=0 \\ 0 & \text{if } N \text{ is odd} \\ \dfrac{(-1)^n q^{n(n-1)/2} (q^{r+1})_{n-1} (1-q^{r+2n})}{(q)_n} & \text{if } N = 2n > 0 \end{cases}.$$