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QUADRATIC FORMS, SUMS OF SQUARES, THETA FUNCTIONS AND INTEGRAL LATTICES CONFERENCE

March 11 – 14, 2009

University of Florida, Gainesville, FL 32611

ABSTRACTS

George Andrews, Pennsylvania State University
11:00AM - 11:50AM, THURSDAY, MARCH 12

Parity in Partition Identities
Parity has played a substantial role in partition identities from Euler onward. In the last few decades, studies beginning with the Göllnitz-Gordon identities and continuing with the work of Alladi have continued these developments. In this talk, we shall discuss several new discoveries. First we look at the appearances of even parts in the Rogers-Ramanujan-Gordon identities. We shall also introduce "parity indices" which not only interact with the Rogers-Ramanujan identities but also throw light on mock theta functions. Special attention will be given to the role played by the little q-Jacobi polynomials.

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Roger Baker, Brigham Young University
10:00AM - 10:20AM, FRIDAY, MARCH 13

The values of quadratic forms at squarefree points
I report on a project to obtain analogs of results of Heath-Brown on the representations of integers by quadratic forms in at least 3 variables. Heath-Brown uses a new form of the Hardy-Littlewood circle method, which he described in a Crelle paper in the nineties, and which has since been applied by himself, Hooley and the speaker. Briefly, one finds an asymptotic formula for the number of representations with smooth weights attached. In this project, all the variables have to be squarefree, which makes matters a bit more complicated. In fact, in the paper I published on this in 2004, there were two cases (representing the value 0, quaternary form with determinant a square; and representing the value 0, ternary form) which I could not push through. I will report on my renewed efforts in these 'harder cases'.

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Bruce Berndt, University of Illinois, Urbana
9:00AM - 9:50AM, SATURDAY, MARCH 14

Ramanujan's Contributions to Theta Functions
We provide a survey of Ramanujan's contributions to the theory and application of theta functions, subject to the limitations of a 50 minute lecture.

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Atul Dixit, University of Illinois at Urbana-Champaign
4:30PM - 4:50PM, FRIDAY, MARCH 13

Analogues of a transformation formula of Ramanujan
In a manuscript in 'The Lost Notebook and Other Unpublished Papers' is present a beautiful transformation formula of Ramanujan involving the Gamma and Riemann zeta functions which can be proved using an identity in Ramanujan's paper 'New expressions for Riemann's functions $\xi(s)$ and $\Xi(t)$'. Here we make use of two other identities given in the same paper to derive two transformation formulas both involving infinite series consisting of Hurwitz zeta functions and bearing a relation to a certain integral involving the Riemann Ξ function.

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Noam Elkies, Harvard University
9:00AM - 9:50AM, WEDNESDAY, MARCH 11

Weighted theta functions, and a coding analog
There is a well-known analogy between the theta function of a lattice and the Hamming weight enumerator of a linear code. Delsarte extended this analogy to theta functions and enumerators with harmonic weights. After reviewing the properties and some uses of harmonically weighted theta functions, we outline a new approach to Delsarte's discrete harmonic polynomials and some of its applications that highlights the code-lattice analogy. This is joint work with Scott Kominers.

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Hershel Farkas, Hebrew University, Jerusalem
10:00AM - 10:20AM, THURSDAY, MARCH 12

Theta Constant Identities and Number Theory
The one dimensional theta constant identities yield information concerning the number of representations of a positive integer as sums of squares or sums of triangular numbers. In this talk we shall show what types of further information we obtain from the higher dimensional theta constant identities. We shall restrict the discussion to dimension 2 but the theory can be easily extended to higher dimensions.

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Amanda Folsom, University of Wisconsin
3:00PM - 3:20PM, FRIDAY, MARCH 13

q-series and weight 3/2 Maass forms
Despite the presence of many famous examples, the precise interplay between basic hypergeometric series and modular forms remains a mystery. We consider this problem for canonical spaces of weight 3/2 harmonic Maass forms. Using recent work of Zwegers, we exhibit forms that have the property that their holomorphic parts arise from Lerch-type series, which in turn may be formulated in terms of the Rogers-Fine basic hypergeometric series. Classical theta functions play a prominent role, and as an application, we relate our results to earlier work of Zagier, Hirzebruch-Zagier, and Eichler, pertaining to Hurwitz class numbers. This is joint work with Kathrin Bringmann and Ken Ono.

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Ryan Skip Garibaldi, Emory University
10:00AM - 10:20AM, WEDNESDAY, MARCH 11

Vanishing of trace forms on Lie algebras
Every representation of an algebraic group G induces a symmetric bilinear form on the Lie algebra Lie(G) called a trace bilinear form. Several theorems regarding such groups G over a field include the hypothesis that there is some representation whose trace bilinear form is nondegenerate. We settle the question of existence of such a representation by reducing it to a problem about representations of G defined over the integers and symmetric bilinear forms on integral lattices.

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Robert Griess, University of Michigan
2:00PM - 2:50PM, THURSDAY, MARCH 12

Barnes-Wall Lattices and their Cousins
The series of Barnes-Wall lattices, of ranks 2^d, are useful in providing context and tools for combinatorial situations, such as spherical codes and constructions of new families of lattices with moderately high minimum norms. The role of finite group theory here is significant. We shall discuss our recent work along these lines.

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Jonathan Hanke, University of Georgia at Athens
11:00AM - 11:50AM, WEDNESDAY, MARCH 11

The 290-Theorem and representing numbers by quadratic forms
This talk will describe several finiteness theorems for quadratic forms, and progress on the question: "Which positive definite integer-valued quadratic forms represent all positive integers?". The answer to this question depends on settling the related question "Which integers are represented by a given

quadratic form?" for finitely many forms. The answer to this question can involve both arithmetic and analytic techniques, though only recently has the analytic approach become practical.
We will describe the theory of quadratic forms as it relates to answering these questions, its connections with the theory of modular forms, and give an idea of how one can obtain explicit bounds to describe which numbers are represented by a given quadratic form.

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Byungchan Kim, University of Illinois, Urbana
4:00PM - 4:20PM, WEDNESDAY, MARCH 11

Combinatorics of partial theta functions
A partial theta function is a sum of the form $\sum_{n=0}^{\infty}(-1)^n q^{n(n-1)/2}x^n$. Combinatorially, identities containing partial theta function are very interesting since they indicate what remains after numerous cancellations of certain kinds of partitions. In this talk, we will discuss the combinatorics of some identities involving partial theta functions in Ramanujan's lost notebook. After this, we define a subpartition of a partition, which is a generalization of the Rogers-Ramanujan subpartition that was introduced by L. Kolitsch. Finally, we will see how subpartitions are related to partial theta functions.

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Kagan Kursungoz, Pennsylvania State University
5:00PM - 5:30PM, THURSDAY, MARCH 12

Parity Considerations in Andrews-Gordon Identities
In a recent paper, Andrews revisited his generating function for $b_{k,a}(m, n)$, the number of partitions of n into m parts, where no consecutive pair of integers together appear more than $k-1$ times, and 1 appears no more than $a-1$ times. He gave further refinements involving parity of occurences of parts. We will give a direct and purely combinatorial construction of the original generating function, and show how the refinements can be alternatively obtained. We will present the proof of a related conjecture of Andrews' as time allows.

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Karl Mahlburg, MIT
10:00AM - 10:20AM, SATURDAY, MARCH 14

Asymptotics for crank and rank moments
(Joint work with K. Bringmann and R. Rhoades). Dyson introduced his famous rank statistic for partitions in order to understand the Ramanujan congruences, and Garvan and Andrews later found the long-sought after crank to complete Dyson's challenge. A number of recent applications, such as Andrews' study of Durfee symbols and the smallest parts partition function, have relied on properties of the moments of the crank and rank functions. Garvan observed and conjectured that the crank moments are always larger than the rank moments. We prove a refined version of this conjecture by

calculating the asymptotic main term for the difference of the crank and rank moments, and observe that it is always positive.

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Guillermo Mantilla, University of Wisconsin
2:00PM - 2:20PM, WEDNESDAY, MARCH 11

Integral trace forms associated to cubic extensions
Given a nonzero integer d we know, by Hermite's Theorem, that there exist only finitely many cubic number fields of discriminant d. A natural question is, how to refine the discriminant in such way that we can tell, when two of these fields are isomorphic. Here we consider the binary quadratic form $q_K : Tr_{K/\mathbb{Q}}(x^2)|_{O_K^0}$, and we show that if d is a positive fundamental discriminant, then the isomorphism class of q_K, as a quadratic form over $\mathbb{Z}^2$, gives such a refinement.

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Riad Masri, University of Wisconsin
4:00PM - 4:20PM, FRIDAY, MARCH 13

Equidistribution of Heegner points and the partition function
In this talk, we will show how the equidistribution of Galois suborbits of Heegner points on the modular curve $X_0(6)$ can be used to obtain a new asymptotic formula for the partition function $p(n)$. We will then show how this formula can be used to significantly sharpen the classical error bounds of Rademacher and D. H. Lehmer. This is joint work with Amanda Folsom.

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Richard McIntosh, University of Regina, Canada
2:30PM - 2:50PM, SATURDAY, MARCH 14

Divisor sums and theta functions
The logarithmic derivative with respect to x of the Jacobi theta function $j(x, q)$ involves the sum $T(x, q)$. If $N^+_{k,m}(n)$ equals the number of positive divisors of n that are congruent to $k \mod m$ and $N^-_{k,m}(n)$ equals the number of negative divisors of n that are congruent to $k \mod m$, then $T(q^k, q^m) = \sum_{n>0}(N^+_{k,m}(n) - N^-_{k,m}(n))q^n$. An outline of a proof that $T(q^k, q^m)$ is a theta function will be given. As defined by Dean Hickerson in his proof of the mock theta conjectures, a theta function is a finite sum of theta products times powers of q.

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Stephen C. Milne, The Ohio State University
11:00AM - 11:50AM, SATURDAY, MARCH 14

Sums of squares, the Leech lattice, and new formulas for Ramanujan's tau function and other classical cusp forms
As motivation, we first recall our infinite families of explicit exact formulas involving either squares or triangular numbers, two of which generalize Jacobi's (1829) 4 and 8 squares identities to $4n^2$ or $4n(n+1)$ squares, respectively, without using cusp forms such as those of Glaisher or Ramanujan for 16 and 24 squares. We derive our formulas by utilizing combinatorics to combine a variety of methods and observations from the theory of Jacobi elliptic functions, continued fractions, Hankel or Turanian determinants, Lie algebras, Schur functions, and multiple basic hypergeometric series related to the classical groups. We also note our derivation proof of the two Kac and Wakimoto (1994) conjectured identities concerning representations of a positive integer by sums of $4n^2$ or $4n(n+1)$ triangular numbers, respectively. These conjectures arose in the study of Lie algebras and have also recently been proved by Zagier using modular forms. Related and subsequent work of Don Zagier, Ken Ono, Getz and Mahlburg, Rosengren, Imamoḡlu and Kohnen, H.-H. Chan and K. S. Chua, and, H.-H. Chan and C. Krattenthaler is very briefly reviewed. We conclude with a discussion of our new formulas for Ramanujan's tau function, including one in terms of the Leech lattice. Utilizing classical elliptic function invariants, we first sketch our derivation of several useful new formulas for Ramanujan's τ function. This work includes: the main pair of new formulas for the τ function that "separate" the two terms in the classical formula for the modular discriminant, a generating function form for both of these formulas, a Leech lattice form of one of these formulas, and a triangular numbers form. We then present analogous new formulas for several other classical cusp forms that appear in quadratic forms, sphere-packings, lattices and groups. If time allows, an additional application to the theory of quadratic forms is also given.

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Gabriele Nebe, RWTH Aachen, Germany
3:00PM - 3:20PM, WEDNESDAY, MARCH 11

Automorphism groups of Type II codes
With my PhD student Annika Günther we showed that the automorphism group of a binary doubly-even self-dual code is always contained in the alternating group using the theory of integral quadratic forms. On the other hand, given a permutation group G of degree n there exists a doubly-even self-dual G-invariant code if and only if n is a multiple of 8, every simple self-dual $\mathbb{F}_2G$-module occurs with even multiplicity in $\mathbb{F}_2^n$, and G is contained in the alternating group.

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Ken Ono, University of Wisconsin
11:00AM - 11:50AM, FRIDAY, MARCH 13

Heegner divisors, L-functions and Maass forms
In the 1980s, Gross and Zagier proved a deep theorem which gave a formula for central derivatives of modular L-functions in terms of the Neron-Tate heights of Heegner points. Also in the early

1980s, Waldspurger proved that the generating function for the central values of quadratic twists is essentially a modular form. It is natural to ask whether these two problems (values and derivatives) can be investigated in a uniform way by combining and extending features of these results. In joint work with Bruinier, we obtain such a result. We show that the Fourier expansions of harmonic Maass forms can be used to study both central values and derivatives of quadratic twists of modular L-functions. The key idea involves the arithmetic of Maass-Heegner points which are associated to canonical quadratic forms.

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Raman Parimala, Emory University
9:00AM - 9:50AM, THURSDAY, MARCH 12

Isotropy of quadratic forms over function fields of p-adic curves
Let K be a field of characteristic not 2. The u-invariant of K, denoted by $u(K)$, is the maximum dimension of anisotropic quadratic forms over K. It remains an open question, whether the finiteness of $u(K)$ implies finiteness of $u(K(t))$ where $K(t)$ denotes the rational function field in one variable over K. This was an open question even when $K = Q_p$ until a decade ago. In analogy with the positive characteristic local field case, the u-invariant of $Q_p(t)$ was expected to be 8. This was proved to be true for p not equal to 2, by Suresh-Parimala. Whether $u(Q_p(t)$ is finite is still an open question We shall discuss in this lecture the history of the problem and a more recent approach to this problem via certain patching techniques for fields due to Harbater-Hartman-Krashen.

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Jeremy Rouse, University of Illinois, Urbana
4:00PM - 4:20PM, THURSDAY, MARCH 12

Bounds for the coefficients of powers of the Δ-function
Let

$$\sum_{n=k}^{\infty} \tau_k(n) q^n = q^k \prod_{n=1}^{\infty} (1 - q^n)^{24k}.$$

Work of Deligne implies that there is a constant C_k so that $|\tau_k(n)| \leq C_k d(n) n^{(12k-1)/2}$. We will show that C_k tends to zero very rapidly as $k \to \infty$.

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Michael Rowell, Pacific University
4:30PM - 4:50PM, SATURDAY, MARCH 14

Obtaining New Indefinite Quadratic Forms
In this talk we will present a brief introduction to indefinite quadratic forms and their connections with partition theory. With the help of a new conjugate Bailey pair, we will consider new proofs of the indefinite quadratic forms and present how this technique can be used to create new similar identities.

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Hideyo Sasaki, Otemae College, Itami, Japan
3:00PM - 3:20PM, THURSDAY, MARCH 12

Quaternary universal forms over $\mathbf{Q}(\sqrt{13})$
Let $F = \mathbb{Q}(\sqrt{m})$ be a real quadratic field over $\mathbb{Q}$ with m a square-free positive rational integer and $\mathcal{O}$ be the integer ring in F. A totally positive definite integral n-ary quadratic form $f = f(x_1, \ldots, x_n) = \sum_{1\le i,j\le n} \alpha_{ij} x_i x_j$ ($\alpha_{ij} = \alpha_{ji} \in \mathcal{O}$) is called universal if f represents all totally positive definite integers in $\mathcal{O}$. Chan-Kim-Raghavan proved that ternary universal forms over F exist if and only if $m = 2$, 3, 5 and determined all such forms. There exists no ternary universal form over real quadratic fields whose discriminants are greater than 12. We give the result that there are only two quaternary universal forms (up to equivalence) over $\mathbb{Q}(\sqrt{13})$. For the proof of universality we apply the theory of quadratic lattices.

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Rainer Schulze-Pillot, Saarland University, Saarbruecken
9:00AM - 9:50AM, FRIDAY, MARCH 13

Representation of quadratic forms
We discuss the recent theorem of Ellenberg and Venkatesh on representation of integral quadratic forms by integral positive definite quadratic forms. We compare it with previous results which use arithmetic methods or analytic methods. We show that it is valid under weaker conditions on the represented form than those originally used and obtain some corollaries.

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James Sellers, Pennsylvania State University
3:00PM - 3:20PM, SATURDAY, MARCH 14

Elementary proofs of parity results for 5–regular partitions
In a paper which appeared in late 2008, Calkin, Drake, James, Law, Lee, Penniston and Radder use the theory of modular forms to examine 5–regular partitions modulo 2 and 13–regular partitions modulo 2 and 3. They obtain and conjecture various results. In this note, we use nothing more than Jacobi's triple product identity to obtain results for 5–regular partitions stronger than those obtained by Calkin and his collaborators. In particular, we find infinitely many Ramanujan–type congruences for $b_5(n)$ in a straightforward manner relying on an easily–proven relationship between $b_5(4n+1)$ and the number of representations of an integer by the quadratic form $2x^2 + 5y^2$. This is joint work with Michael Hirschhorn of the University of New South Wales.

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Andrew Sills, Georgia Southern University
4:00pm - 4:20pm, Saturday, March 14

A new look at some partition identities of Goellnitz
H. Göllnitz is famous for his discovery of several partition identities of the Rogers-Ramanujan-Schur type. We shall discuss some new q-series connections and partition identities inspired by his work.

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Jaebum Sohn, Yonsei University, Korea / University of Illinois
4:30pm - 4:50pm, Wednesday, March 11

Character analogues of theorems of Ramanujan, Koshliakov and Guinand
We derive analogues of theorems of Ramanujan, Koshliakov and Guinand for primitive characters which consist of infinite series of modified Bessel functions. As particular examples, transformation formulas involving the Legendre symbol and sums-of-divisors functions are established.

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John Thompson, University of Florida and Cambdridge
2:00pm - 2:50pm, Friday, March 13

Dirichlet Series and SL(2,Z)
The construction of an elliptic curve of conductor 15 which arises from a representation of $SL(2,\mathbb{Z})$ on the space of Dirichlet series has led directly via the Taniyama-Shimura-Weil theorem to a study of the function field of $\Gamma(15)$. Peter Sin and I continue to examine various aspects of the situation, and I hope to make use of the tree associated to $PSL(2,\mathbb{Z})$ to study the zeroes of the zeta function. The task is daunting and engrossing.

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Heiko Todt, Pennsylvania State University
5:00pm - 5:30pm, Friday, March 13

Asymptotic formulas for partitions into (distinct) k-th powers
E.M. Wright proved a transformation formula for the generating function for partitions into k-th powers, which he then used to find an asymptotic formula for these partitions. I will give a summary of his results and also show how his transformation formula can be applied to partitions into distinct k-th powers as well. In addition a theorem by Meinardus can be applied to find different asymptotic formulas for these partitions, and I will show how the results relate to each other.

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Akshay Venkatesh, Stanford University
5:10pm - 6:00pm, Wednesday, March 11

Geometry of Numbers, Old and New
The geometry of numbers is an old and beautiful method of solving equations in whole numbers using geometrical ideas. I shall discuss this method, illustrating with examples drawn from the theory of quadratic forms. I will then describe some modern descendants of the geometry of numbers; these draw on ideas from other mathematical fields including ergodic theory and harmonic analysis.

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John Voight, University of Vermont
2:30pm - 2:50pm, Wednesday, March 11

Characterizing quaternion rings
We extend the notion of quaternion algebras over fields to an arbitrary commutative ring. We give a characterization of such rings in terms of ternary quadratic modules, extending work of Gross and Lucianovic.

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Ae Ja Yee, Pennsylvania State University
4:30pm - 4:50pm, Thursday, March 12

Parity Problems in Partitions
In his recent paper, George Andrews investigated a variety of parity questions in partition identities. At the end of the paper, he then listed 15 open problems. In this talk, we will discuss Questions 1, 2, 3, 5, 5', 9, and 10 from his list.

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Sander Zwegers, University College Dublin
2:00pm - 2:20pm, Saturday, March 14

Multivariable Appell functions and non-holomorphic modular forms
Multivariable Appell functions are a generalization of both the Appell functions, which play an important role in the theory of Ramanujan's mock theta functions, and of the ordinary theta functions. General results for (normalized) level 1 Appell functions are known: they can be seen as the holomorphic parts of weight 1/2 harmonic Maass forms. For higher level Appell functions the situation is slightly more complicated, but they can still be seen as the holomorphic parts of certain non-holomorphic modular forms. In this talk we'll show how these multivariable Appell functions fit into the same picture.

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