CHAPTER 0

Preliminaries

Throughout, we will use the following standard conventions:

$\mathbb{Z}$ = The ring of integers.

$\mathbb{Q}$ = The field of rational numbers.

$\mathbb{R}$ = The field of real numbers.

$\mathbb{C}$ = The field of complex numbers.

$\mathfrak{O}_{\mathcal{K}}$ = The ring of integers of a number field $\mathcal{K}$.

$[t]$ = The greatest integer less than or equal to t.

$\{t\} = t - [t]$, the fractional part of t.

$\mathcal{H} = \{a + bi \in \mathbb{C} \mid b > 0\}$, the complex upper half plane.

$\mathcal{H}^* = \mathcal{H} \cup \mathbb{Q} \cup \{i\infty\}$.

$x = e^{2\pi i\tau}$, where $\tau \in \mathcal{H}$.

$\eta(\tau) = x^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - x^n)$, the usual Dedekind η-function with $x = e^{2\pi i\tau}$.

$SL_2(\mathbb{Z}) = \left\{ \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \;\middle|\; a, b, c, d \in \mathbb{Z} \text{ and } ad - bc = 1 \right\}$.

$\Gamma = SL_2(Z) \big/ (\pm I)$.

$\Gamma_0(N) = \left\{ \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma \;\middle|\; c \equiv 0 \bmod(N) \right\}$.

$\Gamma_1(N) = \left\{ \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma_0(N) \;\middle|\; a \equiv d \equiv 1 \bmod(N) \right\}$.

$\Gamma^0(N) = \left\{ \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma \;\middle|\; b \equiv 0 \bmod(N) \right\}$.

$X_0(N) = \mathcal{H}^* \big/ \Gamma_0(N)$, the Riemann surface obtained by identifying points of $\mathcal{H}^*$ modulo $\Gamma_0(N)$.

$X_1(N) = \mathcal{H}^* \big/ \Gamma_1(N)$.

$S_k(G, \chi)$= The vector space of cusp forms of weight k on the subgroup $G \subseteq \Gamma$ with character χ. If omitted, χ will be understood to be 1.