UNIVERSITY OF CALIFORNIA

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Arithmetic Properties of modular forms

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Philosophy

in Mathematics

by

Sinai Robins

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The dissertation of Sinai Robins is approved.

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DEDICATION

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PUBLICATIONS AND PRESENTATIONS

On The Lacunarity of η-products, in progress.

The Rotating Coin Puzzle, UCLA Science Journal.

ABSTRACT OF THE DISSERTATION

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Professor Basil Gordon, Chair

This dissertation is an inquiry into arithmetic properties of the Fourier coefficients of modular forms. There are deep properties of many combinatorial identities which one does not see by merely using "elementary" combinatorics and which beg to be dealt with on a more fundamental level. An example is given by the Rogers-Ramanujan identities, where a deeper understanding requires an examination of the Riemann surface on which the relevant forms really live. These forms may have weight 0, non-multiplicative coefficients, and/or possess a complicated multiplier system, so that the usual methods of Swinnerton-Dyer and others may not always apply; nevertheless, the Fourier coefficients still possess many subtle and striking congruential properties.

Many researchers, including Watson, Birch, Bressoud and Biagioli, have investigated Ramanujan's "40 identities", and they have now all been proved. However

the question of how such identities can be discovered has remained largely unexplored. In chapter 1 of this thesis new identities and generalizations are found, geometric motivation for most of them is given, and a technique using modular forms to generate new identities is derived. In investigating these functions, it turns out that we are naturally led to study when functions live on $X_1(N)$, the Riemann surface playing the crucial role in the development of these functional identities. This question is in itself of importance. It is on $X_1(N)$ that we study the properties of these functions, using the Riemann-Roch Theorem to obtain bases for the relevant vector spaces. The main thrust of this portion of the thesis, as opposed to the approach taken by previous researchers, is the systematic discovery and proof of new identities.

In chapter 2 we address the problem of lacunarity, and generalize a 1985 paper of Serre which determines the even powers of the Dedekind η-function which are lacunary. We prove that there are only a finite number of lacunary η-products of certain more general types. One of these types arises from the intimate connection between η-products of the form $\eta(\tau)^r\eta(2\tau)^s$ and affine root systems of Lie algebras, manifested by the Macdonald identities. This fact is one motivation for the study of lacunary forms of the type $f(\tau) = \eta(\tau)^r\eta(2\tau)^s$.