

JAMES MAYNARD TO RECEIVE 2014 SASTRA RAMANUJAN PRIZE

The 2014 SASTRA Ramanujan Prize will be awarded to Dr. James Maynard of Oxford University, England, and the University of Montreal, Canada. The SASTRA Ramanujan Prize was established in 2005 and is awarded annually for outstanding contributions by young mathematicians to areas influenced by the genius Srinivasa Ramanujan. The age limit for the prize has been set at 32 because Ramanujan achieved so much in his brief life of 32 years. The prize will be awarded during December 21-22 at the International Conference on Number Theory at SASTRA University in Kumbakonam (Ramanujan's hometown) where the prize has been given annually.

Dr. Maynard has made spectacular contributions to number theory, especially on some famous problems on prime numbers. The theory of primes is an area where questions which are simple to state can be very difficult to answer. A supreme example is the celebrated "prime twins conjecture" which states that there are infinitely many prime pairs that differ by 2. In the last twelve months, Dr. Maynard has obtained the strongest result towards this centuries old conjecture by proving that the gap between consecutive primes is no more than 600 infinitely often. Not only did he significantly improve upon the earlier path-breaking work of Goldston, Pintz, Yildirim, and Zhang, but he achieved this with ingenious methods which are simpler than those used by others. Maynard's remarkable success on the "small gap problem" on primes is built upon the ideas and results he developed in his doctoral work a few years ago on some other famous problems on primes such as the Brun-Titchmarsh inequality. Maynard's results and methods have led to a resurgence of activity worldwide in prime number theory.

In his doctoral thesis of 2013 at Oxford University written under the direction of Professor Roger Heath-Brown, Maynard obtained a number of deep results on some fundamental problems. One of the intriguing and important questions on prime numbers is how uniformly they are distributed in various arithmetic progressions of integers up to a given magnitude. There is a heuristic estimate for the number of such primes in arithmetic progressions, and one usually gets a bound of the order of magnitude given by the heuristic. This Brun-Titchmarsh problem becomes difficult when the modulus or gap between the members of the progression become very large, and results are weaker compared to the conjectured size. Earlier researchers had relied on certain unproved hypothesis concerning the Siegel zeros of L -functions in order to treat these large moduli. But Maynard showed in his thesis (without appeal to any hypothesis) that the number of primes in arithmetic progressions is bounded by the suspected heuristic size for arithmetic progressions whose moduli do not exceed the eighth root of the largest member of the progression. This important paper appeared in *Acta Arithmetica* in 2013.

A generalization of the prime twins conjecture is the prime k -tuples conjecture which states that an admissible collection of k linear functions will simultaneously take k prime values infinitely often in values of the argument. In the last one hundred years, several partial results towards the k tuples conjecture have been obtained either by replacing prime values of some of these linear functions by "almost primes" (which are integers with a bounded number of prime factors) or by bounding the total number of prime factors in the product of these linear functions. Another major achievement in his doctoral thesis is his work on "Almost-prime k -tuples" (that was published in *Mathematika* in 2014) in which

he obtains bounds for the number of prime factors in the product of these admissible linear functions. These bounds are superior to bounds obtained by earlier researchers, except in the case of the product of three linear functions where his result was just as strong as the 1972 result of Porter who had confirmed that the triple product will have no more than 8 prime factors infinitely often. But in a separate paper that appeared in the Proceedings of the Cambridge Philosophical Society in 2013, Maynard broke the impasse by improving on Porter's result and showing that a product of three admissible linear functions will have no more than 7 prime factors infinitely often. In establishing these fundamental results, Maynard introduced a number of new methods and techniques that enabled him to achieve a sensational result on the small gaps problem on primes within a year of completing his DPhil - when he was a post-doctoral fellow at the University of Montreal, Canada, with Professor Andrew Granville as his mentor.

The Prime Number Theorem implies that the average gap between the n -th prime and the next prime is asymptotic to $\log n$. Two questions arise immediately: (i) How small can this gap be infinitely often (the small gap problem), and (ii) How large can this gap be infinitely often (the large gap problem)? The prime twins conjecture says that the gap is 2 infinitely often.

It was a sensation a few years ago when Goldston, Pintz, and Yildirim (GPY) showed that over a sequence of integers n that tend to infinity, the gap between the n -th prime and the next can be made arbitrarily smaller than the average gap. It was shown by GPY that if a certain conjecture of Elliott and Halberstam on the distribution of primes in arithmetic progression holds, then the gap could be made as small as 16 infinitely often. Two years ago, Y. Zhang stunned the world by showing that infinitely often the smallest gap is no more than 70 million! This was the first time a bounded gap was established unconditionally. For this seminal work on the small gap problem, Goldston, Pintz, Yildirim, and Zhang received the 2014 Cole Prize of the American Mathematical Society.

Zhang's method was quite complex. He had to circumvent the use of a hypothesis that went beyond the Bombieri-Vinogradov Theorem. Terence Tao by leading the polymath project, reduced Zhang's bound to 4680. But within a few months of Zhang's achievement, Maynard took the world by storm by establishing in a simpler and more elegant fashion, that the gap between the primes is no more than 600 infinitely often! This sensational paper on "Small gaps between primes" will soon appear in the Annals of Mathematics. Actually in this paper Maynard establishes a number of other deep results. For example he shows that for any given integer m , the gap between the $n + m$ -th prime and the n -th prime is bounded by a prescribed function of m . Very recently, Manard has joined the Polymath project and now the gap between consecutive primes has been shown to be no more than 246 infinitely often by adapting the method in Maynard's paper in the Annals of Mathematics. This is to appear in the journal Research in the Mathematical Sciences.

Within the last month, Maynard has announced a solution of the famous \$10,000 problem of Paul Erdős concerning large gaps between primes. This was also simultaneously announced by Kevin Ford, Ben Green, Sergei Konyagin, and Terence Tao, but Maynard's methods are different and simpler. In 1938 Robert Rankin established a lower bound for infinitely many large gaps which remained for many years as the best result on the large gap problem. The great Hungarian mathematician Paul Erdős asked whether the

implicit constant in Rankin's lower bound could be made arbitrarily large, and offered \$10,000 to settle this question either in the affirmative or in the negative. In the last fifty years there have been improvements made by various leading researchers on the value of the implicit constant in Rankin's bound - by Rankin himself in 1962, by Helmut Maier and Carl Pomerance in 1990, and by Janos Pintz in 1997 - but the Erdős problem still remained unsolved. Maynard has now shown that the implicit constant in Rankin's lower bound could be made arbitrarily large. So in settling this notoriously difficult problem, an impasse of several decades has been broken. Another fundamental recent work of Maynard is collaboration with William Banks, and Tristan Freiburg on the limit points of the values of the ratio of the gap between consecutive primes and the average gap. Results on the small gaps problem imply that 0 is a limit point, and the work on the large gap problem shows that ∞ is a limit point. No other limit point is known. In this joint paper it is shown that if 50 random real numbers are given, then one at least of the 50 consecutive gaps between them and 0 will occur as a limit point. Thus in the last three years, Maynard has obtained several spectacular results in the theory of primes by methods which will have far reaching implications.

James Maynard was born in Chelmsford, England on June 10, 1987. He obtained his BA and Masters in Mathematics from Cambridge University in 2009. He then joined Balliol College, Oxford University, where he received his Doctorate in Philosophy in 2013. During 2013-14 he was a post-doctoral fellow at the University of Montreal, Canada. Thus within a period three years, Maynard has startled the mathematical world with deep results in rapid succession. The SASTRA Ramanujan Prize will be his first major award in recognition of his many fundamental results in prime number theory.

The 2014 SASTRA Ramanujan Prize Committee consisted of Professors Krishnaswami Alladi - Chair (University of Florida), Roger Heath-Brown (Oxford University), Winnie Li (The Pennsylvania State University), David Masser (University of Basel), Barry Mazur (Harvard University), Peter Paule (Johannes Kepler University of Linz), and Michael Rapoport (University of Bonn). Previous winners of the Prize are Manjul Bhargava and Kannan Soundararajan in 2005 (two full prizes), Terence Tao in 2006, Ben Green in 2007, Akshay Venkatesh in 2008, Kathrin Bringmann in 2009, Wei Zhang in 2010, Roman Holowinsky in 2011, Zhiwei Yun in 2012, and Peter Scholze in 2013. The award of the 2014 SASTRA Ramanujan Prize to James Maynard is a fitting recognition in the tenth year of this prize and in keeping with the tradition of recognizing path-breaking work by young mathematicians.

Krishnaswami Alladi

Chair: SASTRA Ramanujan Prize Committee