

MATOMÄKI AND RADZIWILL TO RECEIVE 2016 SASTRA RAMANUJAN PRIZE

The 2016 SASTRA Ramanujan Prize will be jointly awarded to Dr. Kaisa Matomäki of the University of Turku, Finland, and Dr. Maksym Radziwill of McGill University, Canada, and Rutgers University, USA. Their recent revolutionary collaborative work on multiplicative functions in short intervals has shocked the mathematical community by going well beyond what could be proved previously even assuming the Riemann hypothesis, and has opened the door to a series of breakthroughs on some notoriously difficult questions such as the Erdős discrepancy problem and Chowla's conjecture, previously believed to be well beyond reach. The SASTRA Ramanujan Prize was established in 2005 and is awarded annually for outstanding contributions by young mathematicians to areas influenced by the genius Srinivasa Ramanujan. The age limit for the prize has been set at 32 because Ramanujan achieved so much in his brief life of 32 years. The prize will be awarded during December 21-22, 2016, at the International Conference on Number Theory at SASTRA University in Kumbakonam (Ramanujan's hometown) where the prize has been given annually. Drs. Matomäki and Radziwill will share the \$10,000 prize this year. They are especially recognized for their spectacular collaboration, and also for their very significant individual contributions.

Kaisa Matomäki is one of the strongest young analytic number theorists in the world today. She made a prominent entrance to the world stage during 2007-09 when she established a number of significant results which are contained in about ten excellent research papers as well as in her outstanding PhD thesis of 2009 submitted to the University of London under the direction of Professor Glyn Harman. Matomäki has about thirty first rate publications on various central questions in number theory, and in many of these papers she improves significantly on the earlier works of some of the most eminent number theorists. These papers span several fields - classical analytic number theory, sieve theory, the theory of modular forms, and Diophantine approximation - thereby showing her versatility and power. She also has several major collaborations with leading number theorists; her outstanding collaboration with Radziwill began in 2014.

Maksym Radziwill is one of the very best of the new generation of analytic number theorists, highly original, and technically one of the strongest and broadest. As an undergraduate at McGill University during 2006-09, he wrote an incredible undergraduate thesis on large deviations of additive functions under the direction of Professor Andrew Granville, thereby demonstrating his superior technical strength at so early a stage of his career. Radziwill burst onto the scene in full force in 2012 and within four years has fundamentally changed several parts of analytic number theory. His exceptional PhD thesis at Stanford University submitted in 2013 was written under the direction of Professor Kannan Soundararajan who won the very first SASTRA Ramanujan Prize in 2005. His astonishingly high rate of productivity is seen by the publication of more a dozen highly original papers on a variety of deep problems in probabilistic number theory, the theory of the Riemann zeta function and more generally of L -functions, as well as the theory of modular forms and elliptic curves. He too has forged major collaboration with several very active number theorists. His most spectacular collaboration is with Matomäki, and indeed this is now viewed as revolutionary.

The Matomäki-Radziwill collaboration which began in 2014 first resulted in a paper

that appeared in 2015 in Geometric and Functional Analysis on the topic of sign changes of Hecke eigenvalues. This really was the starting point of their subsequent joint work on multiplicative functions, and the theorems on this subject established in their paper in the Annals of Mathematics in 2016 are described by top mathematicians as simply stunning. The paper concerns the behavior of multiplicative functions on short intervals, and especially that of the Liouville lambda function which takes value 1 at an integer with an even number of prime factors (counted with multiplicity) and value -1 at an integer with an odd number of prime factors. It is well known that the statement that the lambda function takes values 1 and -1 with asymptotically equal frequency is equivalent to the famous Prime Number Theorem. More refined statements concerning the relative error in this equal frequency are related to the celebrated Riemann Hypothesis, perhaps the most important unsolved problem in mathematics. Such equal distribution results were also known for short intervals, where by short one means intervals of type $[x, x + h]$, with h being a fractional power (< 1) of x . One would expect that such equal frequency would hold when h is of size x to an arbitrarily small positive power, but even with the Riemann Hypothesis, the best that is known is that powers larger than $1/2$ will work. Instead of asking for equal frequency in every short interval of length h , if we require that equal frequency should hold for “almost all” short intervals of length h , then one can reduce the size of h considerably: it was shown that h can be made essentially of size of the $1/6$ -th power of x , and assuming the Riemann Hypothesis, Gao showed that h can be taken as small as a power of $\log x$. Matomäki and Radziwiłł shocked the world by showing unconditionally that equal frequency holds almost always as long as h tends to infinity with x . The ideas introduced by Matomäki and Radziwiłł in achieving this are expected to transform the subject in a major way. Since their paper in the Annals of Mathematics deals more generally with multiplicative functions, there are many other significant implications, such as for example to the distribution of smooth numbers in short intervals.

Smooth numbers are those which are devoid of large prime factors and they figure prominently in the study of factorization algorithms. The problem of the distribution of smooth numbers in short intervals is important and challenging. A corollary of their main theorem on multiplicative functions in short intervals is that given any small positive quantity ϵ , there exists a constant $C(\epsilon)$ depending only on ϵ such that every short interval $[x, x + h]$ of length $h = C(\epsilon)\sqrt{x}$ will contain a smooth number whose prime factors are all less than x raised to power ϵ . Previously such a result on smooth numbers was only known conditionally - that is assuming the Riemann Hypothesis - and it is remarkable that Matomäki and Radziwiłł established this unconditionally.

Sarvadaman Chowla had conjectured that if any k collection of values 1 and -1 are given in any order, then the lambda function will take that sequence of values at k consecutive integers with asymptotic frequency $1/2^k$. This conjecture is yet unsolved. But very recently Matomäki, Radziwiłł and 2006 Fields Medallist and SASTRA Ramanujan Prize Winner Terence Tao have proved that when $k = 3$, each of the eight sign choices occurs with positive proportion (probability). Previously Hildebrand in 1986 had shown that each of these eight sign patterns occur infinitely often. Matomäki, Radziwiłł, and Tao, utilize a combination of Hildebrand’s ideas with the powerful new techniques developed in the Matomäki-Radziwiłł paper in the Annals of Mathematics.

The citation for the 2016 SASTRA Ramanujan Prize is as follows: “Kaisa Matomäki and Maksym Radziwill are jointly awarded the 2016 SASTRA Ramanujan Prize for their deep and far reaching contributions to several important problems in diverse areas of number theory and especially for their spectacular collaboration which is revolutionizing the subject. The prize recognizes that in making significant improvements over the works of earlier stalwarts on long standing problems, they have introduced a number of innovative techniques. The prize especially recognizes their collaboration starting with their 2015 joint paper in *Geometric and Functional Analysis* which led to their 2016 paper in the *Annals of Mathematics* in which they obtain amazing results on multiplicative functions in short intervals, and in particular a stunning result on the parity of the Liouville lambda function on almost all short intervals - a paper that is expected to change the subject of multiplicative functions in a major way. The prize notes also the very recent joint paper of Matomäki, Radziwill and Tao announcing a significant advance in the case $k = 3$ towards a conjecture of Chowla on the values of the lambda function on sets of k consecutive integers. Finally the prize notes, that Matomäki and Radziwill, through their impressive array of deep results and the powerful new techniques they have introduced, will strongly influence the development of analytic number theory in the future.”

Kaisa Matomäki was born in Nakkila, Finland, on 30th April 1985. She attended high school in Valkeakoski, Finland and won the First Prize in the national mathematics competition for Finnish high school students. She did her Masters at the University of Turku and received the Ernst Lindelöf Award for the best Masters Thesis in Finland in 2005. After completing her PhD at the Royal Holloway College of the University of London in 2009 under the direction of Professor Glyn Harman, she returned to Turku where she is an Academy Research Fellow.

Maksym Radziwill was born in Moscow, Russia on 24 February, 1988. In 1991 his family moved to Poland and then in 2006 to Canada. After completing his undergraduate studies at McGill University in 2009 and writing a superb undergraduate thesis there under the guidance of Professor Andrew Granville, he joined Stanford University to work for his PhD with Professor Kannan Soundararajan. After receiving his PhD in 2013, he was a Visiting Member at the Institute for Advanced Study in Princeton in 2013-14. He then took up the Hill Assistant Professorship at Rutgers University for the period 2014-17, and this year he joined the faculty of McGill University as a tenure-track assistant professor.

The 2016 SASTRA Ramanujan Prize Committee consisted of Professors Krishnaswami Alladi- Chair (University of Florida), Henri Darmon (McGill University), Winfried Kohnen (University of Heidelberg), Hugh Montgomery (University of Michigan), Peter Sarnak (Princeton University and the Institute for Advanced Study, Princeton), Michael Schlosser (University of Vienna), and Cameron Stewart (University of Waterloo). Previous winners of the Prize are Manjul Bhargava and Kannan Soundararajan in 2005 (two full prizes), Terence Tao in 2006, Ben Green in 2007, Akshay Venkatesh in 2008, Kathrin Bringmann in 2009, Wei Zhang in 2010, Roman Holowinsky in 2011, Zhiwei Yun in 2012, Peter Scholze in 2013, James Maynard in 2014, and Jacob Tsimerman in 2015. The award of the 2016 SASTRA Ramanujan Prize to Kaisa Matomäki and Maksym Radziwill recognizes two brilliant young mathematicians whose individual contributions and their collaborative work are revolutionizing the field of analytic number theory.