

## ABSTRACTS

**George E. Andrews**, Pennsylvania State University

4:05 - 4:55PM, FRIDAY, MARCH 21

*Reflections on the Rogers-Ramanujan Identities in Statistical Mechanics*

In this talk, I shall describe my initiation by Rodney Baxter into the world of exactly solved models. Much of the talk will examine how the first discoveries were made that led to today's flourishing interaction between  $q$ -series and statistical mechanics. I begin with the events of 1979 surrounding the original hard hexagon model, then move to 1983 for the eight-vertex SOS model and finally to the 1986 lattice gas generalization of the hard hexagon model. The perspective throughout will be that of a somewhat diffident number theorist who was totally amazed that his beloved Rogers-Ramanujan identities possessed importance so far afield from number theory. The impact of physics on the direction of research in partitions and  $q$ -series will be emphasized.

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**Jinho Baik**, Princeton University and University of Michigan, Ann Arbor

3:30 - 3:50PM, SATURDAY, MARCH 22

*Limiting distribution of random growth models*

Many one spatial dimension random growth models are believed to be in the KPZ universality class. Especially the height fluctuation exponent of the models in this class is believed to be  $1/3$ . For a polynuclear growth model, this exponent is proved to be true. Moreover the limiting distribution of the height fluctuation is obtained for this special model. It turns out that limiting distributions are more subtle, and different symmetry of the model yields different distribution. Some of the distributions are related to the largest eigenvalue of large random Hermitian matrix. The polynuclear growth model is in bijection to a longest increasing subsequence of random permutation, and we analyze the limiting distribution of longest increasing subsequence using Toeplitz determinant formula.

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**Murray Batchelor**, ANU, Canberra

1:30 - 1:50PM, FRIDAY, MARCH 21

*The XXZ spin chain and combinatorics*

I shall describe the recently discovered connections between the XXZ Heisenberg spin chain and combinatorics. Here the perspective is from researchers in the world of exactly solved models who have glimpsed the amazing world of alternating sign matrices and related combinatorial objects.

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**Robert Calderbank**, AT&T Labs

4:30 - 5:20PM, SATURDAY, MARCH 22

*Combinatorics, Quantum Computers and Cellular Phones*

This talk explores the connection between quantum error correction and Wireless systems that employ multiple antennas at the base station and the mobile terminal. The two topics have a

common mathematical foundation, involving orthogonal geometry - the combinatorics of binary quadratic forms. We explain these connections, and describe how the wireless industry is making use of a mathematical framework developed by Radon and Hurwitz about a hundred years ago.

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**Hilary Carteret**, University of Waterloo

2:30 - 2:50PM, FRIDAY, MARCH 21

*The discrete quantum walk on the infinite line*

Quantum walks are of interest because of their role in the theory of quantum algorithms. This talk is concerned with the discrete quantum walk on the infinite line. Previous analyses of this system have employed two distinct methods, one based on the Schrodinger formulation of quantum mechanics, and one based on the Path Integral approach. Both of these methods have made significant progress but to date, neither has been able to offer a complete analysis. This talk will outline a new method for analysing these systems (also based on the Path Integral approach) which does give complete and uniformly convergent asymptotics for these systems. This new approach has the additional benefit of being rather easier to use. It also sheds light on the mathematical relationship between the two approaches. We already knew they must be related from Feynman's original arguments for the equivalence of these approaches, but no details of that relationship have been given before for these systems.

This is joint work with Bruce Richmond and Mourad Ismail

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**Philippe Di Francesco**, CEA Saclay

10:50 - 11:40AM, FRIDAY, MARCH 21

*Geometrically constrained statistical models on fixed and random lattice: from hard squares to meanders*

We present various approaches to exact and asymptotic enumeration problems involving configurations of mutually excluding ("hard") particles or polymers on fixed or random lattices. We use two different approaches, based on matrix integrals and field theory of two-dimensional quantum gravity. This leads to the exact "gravitational" solution of the hard square model, and to the asymptotic solution of the meander problem of enumerating non-crossing closed curves crossing a line through  $2n$  points up to homotopy.

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**Daniel Gottesman**, Berkeley and Perimeter Institute, Waterloo

9:50 - 10:40AM, FRIDAY, MARCH 21

*Private Key and Public Key Quantum Cryptography*

Information has always been valuable, never more so than in recent decades, and throughout history people have turned to cryptography in an attempt to keep important information secret. Coherent manipulation of quantum states promises to rearrange the lists of possible and impossible cryptographic tasks. I will describe Shor's algorithm to break classical codes with a quantum computer and protocols for quantum key distribution using single photons to perform unbreakable encryption.

One useful classical technique is the idea of a public key, which can be safely handed out even to an adversary. I will also show how to create quantum protocols with similar properties, and how they can be used to create unbreakable digital signatures.

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**Tony Guttmann**, University of Melbourne

2:00 - 2:20PM, FRIDAY, MARCH 21

*Self-avoiding walks on non-Euclidean, irregular and quasiperiodic lattices*

While the behaviour of two-dimensional self-avoiding walks on regular lattices is well understood, and the critical exponents are believed to be exactly known, far less is known about the behaviour of SAW on semi-regular lattices, on non-Euclidean lattices, and on quasiperiodic tilings. For hyperbolic lattices, exact and numerical studies are discussed which show that the critical exponent  $\gamma = 1$ . For SAW on the  $(3.12^2)$  and  $(4.8^2)$  we find the critical exponent is the same as for regular lattices, and give some exact results for the connective constant on one lattice. Finally, recent extensive numerical work on SAW on Ammann-Beenker tilings and Penrose tilings leads to the conclusion that the critical exponent is the same as for regular lattices.

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**Mourad E.H. Ismail**, University of South Florida

2:00 - 2:20PM, SUNDAY, MARCH 23

*Bethe Ansatz Equation of XXZ Model*

This is joint work with S. S. Lin and S. S. Roan. In this article we have discovered a close relationship between the algebraic Bethe Ansatz of the spin  $s$  XXZ model of a finite size and the  $q$ -Sturm-Liouville problem. We have demonstrated that solutions of the algebraic Bethe Ansatz give rise to the polynomial solutions of a second order  $q$ -difference equation in terms of Askey-Wilson operator. The more general form of algebraic Bethe Ansatz and the mathematical problems relevant to the physics of XXZ model are discussed. Furthermore, the similar correspondence between Bethe Ansatz of XXX model and the Sturm-Liouville type difference equation in terms of the Wilson operator has also been found.

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**Michio Jimbo**, University of Tokyo

9:00 - 9:50AM, SUNDAY, MARCH 23:

*Physical Combinatorics II: Counting form factors*

We revisit the issue of counting all local fields of the restricted sine-Gordon model, in the case corresponding to a perturbation of minimal unitary conformal field theory. The problem amounts to the study of a quotient of certain space of polynomials which enter the integral representation for form factors. This space may be viewed as a  $q$ -analog of the space of conformal coinvariants associated with  $U_q(\widehat{\mathfrak{sl}}_2)$  with  $q = \sqrt{-1}$ . We prove that its character is given by the restricted Kostka polynomial multiplied by a simple factor. As a result, we obtain a formula for the truncated character of the total space of local fields in terms of the Virasoro characters.

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**Pierre Mathieu**, Laval University, Quebec

3:00 - 3:20PM, SUNDAY, MARCH 23

*Summing 'jagged partitions' with exclusion*

By 'jagged partitions' we refer to a collection of non-negative integers  $(n_1, n_2, \dots, n_m)$  with  $n_m \geq 1$  subject to the weakly decreasing conditions  $n_i \geq n_{i+1} - 1$  and  $n_i \geq n_{i+2}$ . The exclusion refers to the following difference condition:  $n_i \geq n_{i+K-1} + 1$  or  $n_i = n_{i+K-1}$  and  $n_{i+1} = n_{i+K-2} + 2$ . We present their generating function. It is a rather direct generalization of that of partitions  $(\lambda_1, \lambda_2, \dots, \lambda_m)$  subject to the difference condition  $\lambda_i \geq \lambda_{i+k-1} + 2$  found by Andrews. These restricted partitions arise in the following physical context: for  $K = 2k$ , they describe the quasi-particle basis of a two-dimensional conformal field theory, namely the graded parafermionic theory with  $\mathbb{Z}_{2k}$  cyclic symmetry.

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**Barry McCoy**, Stony Brook

8:30 - 9:20AM, FRIDAY, MARCH 21

*The 8 vertex-chiral Potts analogy*

We present recent results on the symmetry of the 6 and 8 vertex models at roots of unity and their relation with the chiral Potts model.

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**Stephen Milne**, Ohio State University

3:00 - 3:20PM, SATURDAY, MARCH 22

*New formulas for Ramanujan's tau function and other classical cusp forms*

Utilizing classical elliptic function invariants, we first sketch our derivation of several useful new formulas for Ramanujan's tau function. This work includes: the main pair of new formulas for the tau function that "separate" the two terms in the classical formula for the modular discriminant, a generating function form for both of these formulas, a Leech lattice form of one of these formulas, and a triangular numbers form. We then present analogous new formulas for several other classical cusp forms that appear in quadratic forms, sphere-packings, lattices, and groups.

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**Tetsuji Miwa**, RIMS, Kyoto

2:00 - 2:50PM, SATURDAY, MARCH 22

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**Thomas Prellberg**, Technische Universität Clausthal

3:30 - 3:50PM, SUNDAY, MARCH 23

*A proof of the Monotonicity Conjecture by Friedman, Joichi, and Stanton*

Consider the collection of all integer partitions whose part sizes lie in a given set. The sets for which the generating function has weakly increasing coefficients can be classified subject to a long-standing open conjecture. By proving this conjecture, we can present a complete answer to this problem.

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**James Propp**, University of Wisconsin

11:30 - 12:20PM, SATURDAY, MARCH 22

*q versus lambda: plane partitions, alternating sign matrices, and lattice models*

Most of the talk will be devoted to the relationship between the  $q$  in MacMahon's formula for enumeration of plane partitions and the  $\lambda$  in the Robbins-Rumsey generalization of Dodgson's determinant condensation algorithm. In particular, I will explain how both  $q$  and  $\lambda$ , in different contexts, describe the imposition of a spatially-varying external field on a two-dimensional dimer model with special boundary conditions. I will also show how the discrete Hirota equation provides a different way of putting  $q$  and  $\lambda$  into a common setting. I will conclude by discussing alternating-sign matrices, the six-vertex model, and the fully-packed loop model.

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**Margaret A. Readdy**, University of Kentucky

3:00 - 3:20PM, FRIDAY, MARCH 21

*The pre-WDVV ring of physics and its topology*

We study a simplicial complex arising from the WDVV (Witten-Dijkgraaf-Verlinde-Verlinde) equations of string theory. Using discrete Morse theory, we show the pre-WDVV complex  $\Delta_n$  is homotopy equivalent to a wedge of  $(n-2)!$  spheres of dimension  $n-4$ . We also show the pre-WDVV complex is Cohen-Macaulay.

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**Michael Rubinstein**, AIM, Palo Alto

10:20 - 11:10AM, SUNDAY, MARCH 23

*The Riemann Hypothesis and Random Matrix Theory*

In 1972, it was discovered that the Riemann Zeta function behaves statistically like the characteristic polynomial of a large unitary matrix. Since then, various matrix models have been used with stunning success to make hitherto unimaginable predictions concerning the zeros and values of  $L$ -functions. These results, which I will discuss, confirm the Polya Hilbert Philosophy — that the Riemann Hypothesis is true because the zeros of the Riemann Zeta function somehow correspond to the eigenvalues of a unitary operator.

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**Anne Schilling**, University of California at Davis

2:30 - 2:50PM, SUNDAY, MARCH 23

*Virtual Kleber algorithm*

Kirillov and Reshetikhin conjectured what is now known as the fermionic formula for the decomposition of tensor products of certain finite dimensional modules over quantum affine algebras. This formula can also be extended to the case of  $q$ -deformations of tensor product multiplicities as recently conjectured by Hatayama et al.. From the physics perspective, fermionic formulas are desirable as they reflect the particle content of the underlying model. In its original formulation it is difficult to compute the fermionic formula efficiently. Kleber found an algorithm for the simply-laced algebras which overcomes this problem. We present a method which reduces all other cases to the simply-laced case using embeddings of affine algebras. This is the fermionic analogue of the virtual crystal construction, which is the realization of crystal graphs for arbitrary quantum affine algebras in terms of those of simply-laced type.

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**Vyacheslav P. Spiridonov**, JINR, Dubna

11:30 - 11:50AM, SUNDAY, MARCH 23

*Theta Hypergeometric Series and Integrals*

A general approach to special functions of hypergeometric type associated with Jacobi theta functions is outlined. The elliptic beta integral, proved by the author a few years ago, and its various multivariable generalizations associated with root systems are briefly described.

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**Xavier G. Viennot**, Universite Bordeaux 1

10:20 - 11:10AM, SATURDAY, MARCH 22

*Heaps of pieces in physics*

The notion of “heaps of pieces” has been introduced by the author in 1985, as a “geometrization” of the algebraic notion of commutation monoids defined by Cartier and Foata. The theory has been developed by the Bordeaux group of combinatorics, with strong interaction with statistical mechanics. Recently heaps of pieces has been used again by physicists for the resolution of some models in 2D Lorentzian quantum gravity (Di Francesco et al).

We begin by stating three basic lemma of the theory: an “inversion lemma” giving generating functions of heaps as the quotient of two alternating generating functions of “trivial” heaps, the “logarithmic lemma”, and the “path lemma” saying that any path can be put in bijection with a heap. Many results and explicit formulae or identities in various papers scattered in the combinatorics and physics literature can be unified and viewed as consequence of these three basic lemma, once the translation of the problem into heaps methodology has been made.

The first interaction is with the now classical directed animal models and gas models with hard core interaction, such as Baxter’s hard hexagons model. Combinatorial interpretation of the density of the gas is given (as power series in the variable fugacity), relating the model with directed animals (as given by Dhar,...). Very recently, the “multidirected” animal problem was solved (Bouquet-Melou, Rechnitzer) using heaps methodology.

The second topic is a unifying explanation of the appearance of some  $q$ -Bessel functions in two lattice models: the staircase polygons (or parallelogram polyominoes) (Bender, Delest, Fedou, ...) and the Solid-on-Solid model (Owczarek, Prellberg, ...). It is pleasant to remark that by restricting the pieces of the heaps used in this unified interpretation, one gets the Ramanujan continued fraction and also the Andrews interpretation of the reciprocal of Rogers-Ramanujan identities. The subject is also related to the construction of a basis of the classical Temperley-Lieb algebra and the determination of its Hilbert polynomial.

Finally I will explain the recent use of heaps of pieces methodology for solving some Lorentzian quantum gravity models (Ambjorn, Loll, Di Francesco, Guitter, ... and some joint work with W. James). In particular we relate the appearance of Bessel functions in the continuum limit of the model to the above solutions of staircase polygons and Solid-on-Solid models. The multidirected animals mentioned above play a role in this quantum gravity story.

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**Ole Warnaar**, University of Melbourne

12:00 - 12:20PM, SUNDAY, MARCH 23

*Integral representations of multivariable elliptic hypergeometric series*

We present a conjectural multi-dimensional integral representation for elliptic hypergeometric series of Aomoto-Ito-Macdonald type. Some consequences of this conjecture, such as a multivariable elliptic  ${}_{10}\phi_9$  transformation, will be discussed.

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**F.Y. Wu**, Northeastern University

9:00 - 9:50AM, SATURDAY, MARCH 22

*Some recent results in dimer statistics*

This talk reviews some recent new results on close-packed dimers (dominoes) on planar and nonorientable surfaces. We deduce closed-form expressions for dimer generating functions on the Moebius strip and the Klein bottle. The expressions depend explicitly on whether the linear sizes of the lattice are even or odd, and are given in the form of double products and also in terms of generalized Fibonacci numbers. The solutions lead to an extension of the Stanley-Propp reciprocity relation for dimer enumerations and several curious identities connecting results for different embedding lattices. Using a vertex model formulation we deduce solutions for several two-dimensional regular arrays including the triangular, kagome, and the Union-Jack lattices. We also deduce the generating function for close-packed dimers on a rectangular net with a boundary defect.

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