We will prove the following 

$$\sum_{n=0}^{\infty} p(n)\, q^n = 1 + p(1) q^1 + p(2) q^2 + p(3) q^3 + p(4) q^4 + \cdots$$
$$= 1 + q + 2q^2 + 3q^3 + 5q^4 + \cdots$$
$$= \prod_{m=1}^{\infty} \frac{1}{1-q^m} = \frac{1}{(1-q)} \cdot \frac{1}{(1-q^2)} \cdot \frac{1}{(1-q^3)} \cdots$$

for $|q| < 1$ (Euler) (Convention: $p(0)=1$).

For $n \geq 1$,

$$p(n) = p(n-1) + p(n-2) - p(n-5) - p(n-7)$$
$$+ p(n-12) + p(n-15)$$
$$+ \cdots + (-1)^{k+1}\left(p\left(n - \frac{k(3k-1)}{2}\right) + p\left(n - \frac{k(3k+1)}{2}\right)\right)$$
$$+ \cdots$$

(Euler) (Convention: $p(m) = 0$ if $m < 0$).

<u>Example:</u>

$$p(6) = p(6-1) + p(6-2) - p(6-5)$$
$$= p(5) + p(4) - p(1) = 7 + 5 - 1 = 11$$
$$p(7) = p(7-1) + p(7-2) - p(7-5) - p(7-7)$$
$$= p(6) + p(5) - p(2) - p(0)$$
$$= 11 + 7 - 2 - 1$$
$$= 15.$$

<u>Convergence of Infinite Products</u>

<u>Defn</u> $\prod_{n=1}^{\infty}(1+a_n)$ converges if

$\lim_{N\to\infty} \prod_{n=1}^{N}(1+a_n)$ exists and does not equal zero. We say the product converges absolutely if $\prod_{n=1}^{\infty}(1+|a_n|)$ converges.