$$\frac{1}{1-q}\cdot\frac{1}{1-q^2}\cdots\frac{1}{1-q^m}$$

$$= (1+q^1+q^{1+1}+q^{1+1+1}+\cdots)(1+q^2+q^{2+2}+q^{2+2+2}+\cdots)$$
$$(1+q^3+q^{3+3}+q^{3+3+3}+\cdots)\cdots\cdots(1+q^m+q^{m+m}+\cdots)$$

$$= \sum_{n_1=0}^{\infty} q^{n_1} \sum_{n_2=0}^{\infty} q^{2n_2} \cdots \sum_{n_m=0}^{\infty} q^{n_m m} = \sum q^{1\cdot n_1+2n_2+\cdots+m n_m}$$

$$= 1 + q^1 + (q^{1+1}+q^2) + (q^{1+1+1} + q^{1+2} + q^3) + \cdots$$

$$= 1 + \sum_{n=1}^{\infty} p_m(n)\, q^n$$

where $p_m(n)$ = # of solutions of

$$n = a_1+a_2+\cdots+a_k$$

where $m \geq a_1 \geq a_2 \geq \cdots \geq a_k \geq 1$, $1 \leq k \leq m$

= # of partitions of $n$ into parts not exceeding $m$.

<u>Theorem</u>: Let $m \geq 1$. Then for $|q|<1$

$$\sum_{n=0}^{\infty} p_m(n)\, q^n = \left(\frac{1}{1-q}\right)\left(\frac{1}{1-q^2}\right)\cdots\left(\frac{1}{1-q^m}\right)$$

$$= \prod_{k=1}^{m} \frac{1}{1-q^k}$$

and $p_m(n)$ = # of partitions of $n$ into parts not exceeding $m$.

<u>Example</u> Find $p_3(6)$.

$$\sum_{n=0}^{\infty} p_3(n)\, q^n = \left(\frac{1}{1-q}\right)\left(\frac{1}{1-q^2}\right)\left(\frac{1}{1-q^3}\right).$$