MAPLE:

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> x := 1/(1-q)/(1-q^2)/(1-q^3);
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$$x := \frac{1}{(1-q)(1-q^2)(1-q^3)}$$

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> series(x, q, 10);
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$$1 + q + 2q^2 + 3q^3 + 4q^4 + 5q^5 + 7q^6 + 8q^7 + 10q^8 + 12q^9 + O(q^{10}).$$

So $p_3(6) = 7$.

Partitions of 6 into parts $\leqslant 3$:

3 + 3	2 + 2 + 2	1+1+1+1+1+1
3 + 2 + 1	2+2+1+1	
3 + 1+1+1	2+1+1+1+1	

Theorem

Let $p_{m,d}(n)$ = # of partitions of n into parts $\leq m$ and each part occurs at most d times

Then

$$\sum_{n=0}^{\infty} p_{m,d}(n)\, q^n = (1+q^1+q^{1+1}+\cdots+q^{d})(1+q^2+q^{2+2}+\cdots+q^{2d})\cdots(1+q^m+q^{2m}+\cdots+q^{md})$$

$$= \frac{(1-q^{d+1})}{(1-q)}\frac{(1-q^{2d+2})}{(1-q^2)}\cdots\frac{(1-q^{md+m})}{(1-q^m)}$$

$$= \prod_{k=1}^{m}\frac{(1-q^{k(d+1)})}{(1-q^k)}$$

note: $1+x+\cdots+x^d = \dfrac{x^{d+1}-1}{x-1}$