Example: Find G.F. for $p_{3,3}(n)$.

$$\sum_{n=0}^{\infty} p_{3,3}(n) q^n = \prod_{k=1}^{3} \frac{(1-q^{4k})}{(1-q^k)}$$

```
> mul((1-q^(4*k))/(1-q^k), k=1..3);
```

$$\frac{(1-q^4)(1-q^8)(1-q^{12})}{(1-q)(1-q^2)(1-q^3)}$$

```
> normal(%);
```

$$(q^8-q^7+q^6+q^2-q+1)(q^7+q^6+q^5+q^4+q^3+q^2+q+1)(q^3+q^2+q+1)$$

```
> expand(%);
```

```
> sort(%);
```

$$q^{18}+q^{17}+2q^{16}+3q^{15}+3q^{14}+4q^{13}+5q^{12}+5q^{11}+5q^{10}+6q^9+5q^8+5q^7+5q^6+4q^5+3q^4+3q^3+2q^2+q+1$$

Proof of Eulers Theorem:

Let $m \geq 1$. If $n \leq m$ then any partition

$$n = a_1 + a_2 + \cdots + a_k$$

has $a_k \leq a_{k-1} \leq \cdots \leq a_1 \leq n \leq m$, and

$$p(n) = p_m(n).$$