Hence

$$F_m(q) = \sum_{n=0}^{\infty} p_m(n) q^n \leq \sum_{n=0}^{\infty} p(n) q^n \leq F(q).$$

So

$$\lim_{n \to \infty} F_m(q) = F(q) \leq \sum_{n=0}^{\infty} p(n) q^n \leq F(q),$$

and

$$\sum_{n=0}^{\infty} p(n) q^n = F(q) = \prod_{k=1}^{\infty} \frac{1}{1-q^k}$$

for $0 \leq q < 1$. The result can be extended to $|q| < 1$ by analytic continuation.

Formally, a partition π is a k-tuple

$$\pi = (\lambda_1, \lambda_2, \ldots, \lambda_k) \qquad (\text{some } k \geq 1)$$

of positive integers

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k .$$

π is a partition of n if

$$n = \lambda_1 + \lambda_2 + \cdots + \lambda_k .$$

We write $|\pi| := \lambda_1 + \lambda_2 + \cdots + \lambda_k$ (weight of π or sum of parts). Let $\mathcal{P}$ be the set of partitions.

$$p(n) = \sum_{\substack{\pi \in \mathcal{P} \\ |\pi| = n}} 1,$$

and

$$\sum_{n=0}^{\infty} p(n) q^n = \sum_{\pi \in \mathcal{P}} q^{|\pi|}$$

Empty partition $\pi = ()$, $|\pi| = 0$.