Defn Let $H \subset \mathbb{Z}^+$ & $d \geq 1$.

Let $p(\text{`}H\text{'}(\leq d), n) =$ # of partitions of n into parts which are elements of H each part appearing no more than d times.

Theorem

$$\sum_{n=0}^{\infty} p(\text{`}H\text{'}(\leq d), n)\, q^n = \prod_{m \in H} (1 + q^m + q^{2m} + \cdots + q^{dm})$$

$$= \prod_{m \in H} \frac{(1 - q^{(d+1)m})}{(1 - q^m)} \quad \text{for } |q| < 1.$$

~~Theorem~~

~~Let~~

Cor. $$\sum_{n=0}^{\infty} \underbrace{p(\text{`}H\text{'}(\leq 1), n)}_{\#\text{ of partitions of } n \text{ into distinct parts from } H.} q^n = \prod_{m \in H} (1 + q^m), \text{ for } |q| < 1.$$

Theorem (Euler)

Let $p(D, n) =$ # of partitions of n into distinct parts

Let $p(O, n) =$ # of partitions of n into odd parts.

Then

$$p(D, n) = p(O, n) \quad \text{for all } n.$$