Theorem (Glaisher) (1883) Let $d \geq 1$.
Let N_d = set of positive integers not divisible by d.
Let N = set of positive integers.
Then

$$p('N_{d+1}', n) = p('N'(\leq d), n) \quad \text{for all } n.$$

ie The ~~#~~ # of partitions of n whose parts are not divisible by $(d+1)$
= # of partitions of n whose each part occurs $\leq d$ times.

Proof:

$$\sum_{n=0}^{\infty} p('N'(\leq d), n) q^n = \prod_{n=1}^{\infty} \frac{(1-q^{(d+1)n})}{(1-q^n)}$$

$$= \prod_{\substack{n=1 \\ (d+1) \nmid n}}^{\infty} \frac{1}{1-q^n}$$

$$= \sum_{n=0}^{\infty} p('N_{d+1}', n) q^n, \quad \text{for } |q|<1,$$

& the result follows.

Graphical Representation of Partitions

If $n = \lambda_1 + \lambda_2 + \cdots + \lambda_r$, $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_r$ is a partition then we form the <u>Ferrers graph</u> of this partition by rows of $\lambda_1, \lambda_2, \ldots, \lambda_r$ dots: