Case 2 breaks down when there is a node in common & $s(\lambda) = r+1$ & $\partial(\lambda) = r$.

This is

The image $\tilde{\lambda}$ will not be a ptn into distinct parts

$\longleftarrow (r+1) \longrightarrow$

This partition (with r parts) is

$$(r+1) + (r+2) + \cdots (r+r)$$
$$= r^2 + (1 + \cdots + r)$$
$$= r^2 + \tfrac{1}{2} r(r+1) = \frac{3r^2}{2} + \frac{r}{2} = \frac{r}{2}(3r+1).$$

Hence if n is not of the form $\frac{r}{2}(3r \pm 1)$ then there are no exceptions &

$$p_e(D, n) = p_o(D, n).$$

If $n = \frac{r}{2}(3r \pm 1)$ there is one exceptional partition. If r is even this exception has an even # of parts & $p_e(D,n) - p_o(D,n) = 1$.

If r is odd this one exception has odd # of parts &

$$p_e(D,n) - p_o(D,n) = -1.$$

So

$$p_e(D,n) - p_o(D,n) = (-1)^r \quad \text{if} \quad n = \frac{r}{2}(3r \pm 1).$$

$\square$