Cor. (Euler)

Let $n \geq 1$.

$$p(n) - p(n-1) - p(n-2) + p(n-5) + p(n-7) + \cdots + (-1)^m p(n - \tfrac{1}{2}m(3m-1)) + (-1)^m p(n - \tfrac{1}{2}m(3m+1)) + \cdots = 0$$

where $p(j) = 0$ if $j < 0$.

Proof

$$\left(\prod_{n=1}^{\infty} \frac{1}{1-q^n}\right) \prod_{n=1}^{\infty} (1-q^n) = 1 \qquad \text{for } |q|<1.$$

$$1 = (1 + p(1)q^1 + \cdots + p(n)q^n + \cdots)$$
$$(1 - q - q^2 + q^5 + q^7 - + \cdots + (-1)^m q^{\frac{m}{2}(3m-1)} + (-1)^m q^{\frac{m}{2}(3m+1)} + \cdots)$$

Coeff of q^n:

$$0 = p(n) - p(n-1) - p(n-2) + p(n-5) + p(n-7) - + \cdots + (-1)^m p(n - \tfrac{m}{2}(3m-1)) + (-1)^m p(n - \tfrac{m}{2}(3m+1)) + \cdots. \qquad \square$$