Combinatorial Proof of $p(\mathcal{D}, n) = p(\mathcal{O}, n)$ for all n. (26)

~~Suppose~~

~~$n = r_1 a_1 + r_2 a_2 + \dots + r_s a_s$~~

~~is a partition~~

Suppose $\lambda = (\underbrace{a_1, a_1, \dots, a_1}_{r_1}, \underbrace{a_2, \dots, a_2}_{r_2}, \dots\dots, \underbrace{a_s, a_s, \dots a_s}_{r_s})$ (1

is a partition of n into odd parts.

Let $r_i = \sum_j b_{i,j} 2^j$

be the binary expansion of r_i where $b_{i,j} = 0, 1$.

Then

$$n = \sum_{i=1}^{s} a_i r_i$$

$$= \sum_{i=1}^{s} a_i \sum_j b_{i,j} 2^j$$

$$= \sum_{i=1}^{s} \sum_j b_{i,j} 2^j a_i$$

This gives a partition of n into distinct parts with parts $2^j a_i$ (where $b_{i,j} = 0, 1$).

Conversely, given given a ptn

$(c_1, c_2, \dots, c_t)$ into distinct parts

write

each $c_k = 2^{e_k} d_k$ where d_k is odd.