Example (Subbarao)

The number of partitions of n in which each part appears 2, 3 or 5 times equals the number of ptns of n into parts $\equiv$ 2, 3, 6, 9 or 10 mod 12.

Let $a(n) = \#$ of ptns of n into in which each part appears 2, 3 or 5 times.

Let $b(n) = \#$ of ptns of n into parts $\equiv 2, 3, 6, 9$ or $10 \bmod 12$.

For $|q|<1$,

$$\sum_{n=0}^{\infty} a(n) q^n = (1 + q^{1+1} + q^{1+1+1} + q^{1+1+1+1+1})$$

$$\cdot (1 + q^{2+2} + q^{2+2+2} + q^{2+2+2+2+2})$$

$$\cdot (1 + q^{3+3} + q^{3+3+3} + q^{3+3+3+3+3})$$

$$\vdots$$

$$= \prod_{n=1}^{\infty} (1 + q^{2n} + q^{3n} + q^{5n}).$$

$$= \prod_{n=1}^{\infty} \left((1+q^{2n}) + q^{3n}(1+q^{2n}) \right)$$

$$= \prod_{n=1}^{\infty} (1+q^{2n})(1+q^{3n}) = \prod_{n=1}^{\infty} \frac{(1+q^{2n})(1+q^{3n})(1-q^{2n})(1-q^{3n})}{(1-q^{2n})(1-q^{3n})}$$

$$= \prod_{n=1}^{\infty} \frac{(1-q^{4n})(1-q^{6n})}{(1-q^{2n})(1-q^{3n})}$$