$$\frac{q^m}{m(1-q)} < \frac{q^m}{1-q^m} < \frac{q}{m(1-q)},$$

&

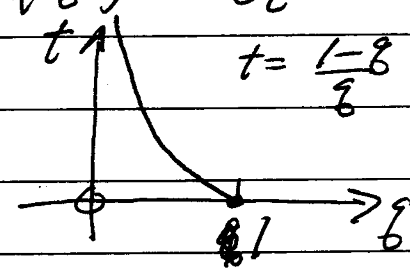
$$\frac{1}{m^2}\frac{q^m}{(1-q)} < \frac{1}{m}\frac{q^m}{1-q^m} < \frac{q}{m^2(1-q)}$$

so

$$\log F(q) = \sum_{m=1}^{\infty} \frac{1}{m}\frac{q^m}{1-q^m} \le \frac{q}{1-q}\sum_{m=1}^{\infty}\frac{1}{m^2}$$

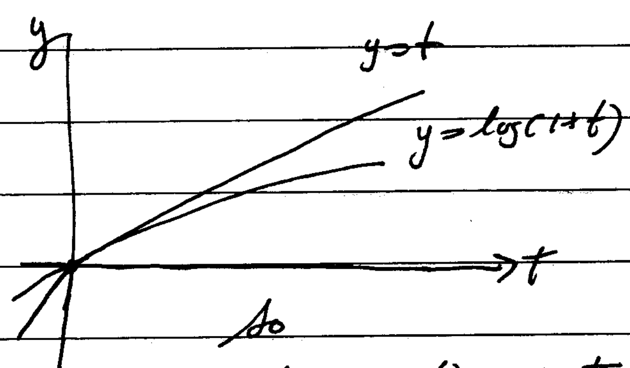
$$= \frac{q}{1-q}\zeta(2) = \left(\frac{q}{1-q}\right)\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{6t}$$

where $t = \frac{1-q}{q} = \frac{1}{q} - 1$



Now,

$$\ln p(n) < \log F(q) - n\log q = \log F(q) + n\ln\frac{1}{q}$$



$$f(t) = -\log(1+t) + t$$

$$f'(t) = 1 - \frac{1}{1+t} = \frac{t}{1+t} > 0$$

for $t > 0$.

So

$$\log(1+t) < t$$

$$1 + t = \frac{1}{q}, \quad \&$$

$$\ln\frac{1}{q} = \ln(1+t) < t$$