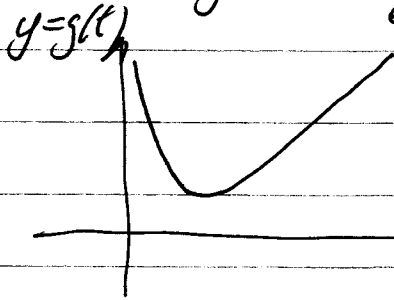


Hence,

$$\begin{aligned} \ln p(n) &< \log F(n) + n \ln \frac{1}{n} \\ &< \frac{\pi^2}{6t} + nt \quad \text{for all } t > 0. \end{aligned}$$

Let  $g(t) = \frac{\pi^2}{6t} + nt \quad t > 0.$



$$g'(t) = n - \frac{\pi^2}{6t^2} = 0$$

where  $n - \frac{\pi^2}{6t^2} = 0$

$$n = \frac{\pi^2}{6t^2} \quad \& \quad nt = \frac{\pi^2}{6t}$$

ie  $nt^2 = \frac{\pi^2}{6}$

$$t = \frac{1}{\sqrt{n}} \frac{\pi}{\sqrt{6}} \quad \& \quad nt = \frac{\pi}{\sqrt{6}} \sqrt{n}$$

$\ln p(n) < 2nt = \frac{2\pi\sqrt{n}}{\sqrt{6}} = \frac{\sqrt{2}}{\sqrt{3}} \pi \sqrt{n}$

&

$$p(n) < e^{\frac{\pi\sqrt{2}}{\sqrt{3}} \sqrt{n}} \quad \square$$