

Example (4) Using MAPLE find formal products for GFs of

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- (i) p(n)
- (ii) p(D,n)
- (iii) The number of partitions of n in which difference between parts is at least 2,

at least up to q^30.

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> with(combinat):
> read "FUNCS.txt":
```

(i)

```
> GFP:=add(P(n)*q^n,n=0..30);
```

$$\begin{aligned}
 \text{GFP} := & 1 + q + 2q^2 + 3q^3 + 5q^4 + 7q^5 + 11q^6 + 15q^7 + 22q^8 + 30q^9 + 42q^{10} + 56q^{11} \\
 & + 77q^{12} + 101q^{13} + 135q^{14} + 4565q^{15} + 176q^{16} + 231q^{17} + 297q^{18} + 385q^{19} \\
 & + 3718q^{20} + 490q^{21} + 627q^{22} + 792q^{23} + 1002q^{24} + 1255q^{25} + 1575q^{26} + 1958q^{27} \\
 & + 2436q^{28} + 5604q^{29} + 3010q^{30}
 \end{aligned}$$

```
> pmake(GFP,q,30);
```

$$\begin{aligned}
 & 1/((1-q)(1-q^2)(1-q^3)(1-q^4)(1-q^5)(1-q^6)(1-q^7)(1-q^8)(1-q^9) \\
 & (1-q^{10})(1-q^{11})(1-q^{12})(1-q^{13})(1-q^{14})(1-q^{15})(1-q^{16})(1-q^{17}) \\
 & (1-q^{18})(1-q^{19})(1-q^{20})(1-q^{21})(1-q^{22})(1-q^{23})(1-q^{24})(1-q^{25}) \\
 & (1-q^{26})(1-q^{27})(1-q^{28})(1-q^{29})(1-q^{30}))
 \end{aligned}$$

(ii)

```
> GFPDP:=add(PDP(n)*q^n,n=0..30);
```

$$\begin{aligned}
 \text{GFPDP} := & 1 + q + q^2 + 2q^3 + 2q^4 + 3q^5 + 4q^6 + 5q^7 + 6q^8 + 8q^9 + 10q^{10} + 12q^{11} \\
 & + 15q^{12} + 18q^{13} + 22q^{14} + 256q^{15} + 27q^{16} + 32q^{17} + 38q^{18} + 46q^{19} + 222q^{20} \\
 & + 54q^{21} + 64q^{22} + 76q^{23} + 89q^{24} + 104q^{25} + 122q^{26} + 142q^{27} + 165q^{28} + 296q^{29} \\
 & + 192q^{30}
 \end{aligned}$$

```
> pmake(GFPDP,q,30);
```

$$\begin{aligned}
 & 1/((1-q)(1-q^3)(1-q^5)(1-q^7)(1-q^9)(1-q^{11})(1-q^{13})(1-q^{15})(1-q^{17}) \\
 & (1-q^{19})(1-q^{21})(1-q^{23})(1-q^{25})(1-q^{27})(1-q^{29}))
 \end{aligned}$$

(iii)

```
> GFPRR:=add(PRR(n)*q^n,n=0..30);
```

$$\begin{aligned}
 \text{GFPRR} := & 1 + q + q^2 + q^3 + 2q^4 + 2q^5 + 3q^6 + 3q^7 + 4q^8 + 5q^9 + 6q^{10} + 7q^{11} + 9q^{12} \\
 & + 10q^{13} + 12q^{14} + 102q^{15} + 14q^{16} + 17q^{17} + 19q^{18} + 23q^{19} + 91q^{20} + 26q^{21}
 \end{aligned}$$