$\boxed{\text{Recurrences for computing } \{a_n\} \,\&\, \{b_n\}}$

Suppose $\{a_n\}_{n=1}^{\infty}, \{b_n\}_{n=1}^{\infty} \subset \mathbb{Z}$ and

$$1 + \sum_{n=1}^{\infty} a_n q^n = \prod_{n=1}^{\infty} (1-q^n)^{b_n} \quad \text{for } |q|<1,$$

and converges absolutely.

Let $A(q) := 1 + \sum_{n=1}^{\infty} a_n q^n$ for $|q|<1$.

Then

$$\log(A(q)) = \log \prod_{n=1}^{\infty} (1-q^n)^{b_n} = \sum_{n=1}^{\infty} b_n \log(1-q^n)$$

$$\frac{d}{dq} \log(A(q)) = \frac{d}{dq}\left(\sum_{n=1}^{\infty} b_n \log(1-q^n)\right)$$

$$\frac{A'(q)}{A(q)} = \sum_{n=1}^{\infty} \frac{(-b_n) n q^{n-1}}{1-q^n}$$

$$q A'(q) = -\left(\sum_{n=1}^{\infty} b_n \frac{n q^n}{1-q^n}\right) A(q)$$

$$\frac{q^n}{1-q^n} = \sum_{m=1}^{\infty} q^{mn} \quad \text{and}$$

$$\sum_{n=1}^{\infty} b_n \frac{n q^n}{1-q^n} = \sum_{n=1}^{\infty} b_n \sum_{m=1}^{\infty} n q^{mn}$$

$$= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} n b_n q^{mn} = \sum_{N=1}^{\infty} \left(\sum_{mn=N} n b_n\right) q^N$$

$$= \sum_{N=1}^{\infty} D_N q^N \quad \text{where} \quad D_N = \sum_{d \mid N} d\, b_d .$$