Corollary Let $n \geq 1$,

$$n\, p(n) = \sum_{j=1}^{n} \sigma(j)\, p(n-j)$$

where $\sigma(j) = \sum_{d|j} d$.

Proof Let $a_n = p(n)$ for $n \geq 1$. Then

$$b_n = -1 \quad \text{for } n \geq 1.$$

$$1 + \sum_{n=1}^{\infty} p(n) q^n = \prod_{n=1}^{\infty} (1-q^n)^{-1}.$$

$$D_j = \sum_{d|j} d\, b_d = -\sum_{d|j} d = -\sigma(j),$$

and by (i),

$$n a_n = -\sum_{j=1}^{n} D_j\, a_{n-j} \quad \&$$

$$n\, p(n) = \sum_{j=1}^{n} \sigma(j)\, p(n-j). \quad \square$$